**Fundamental Theorem of Calculus (FTC)**

FTC Part 1: If $f(x)$ is continuous over an interval $[a, b]$, then over $[a, b]$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\underbrace{\int_a^x f(t)\,\mathrm{d}t}_{=F(x)}\right) = f(x)$$

FTC Part 2: If $f(x)$ is continuous over the interval $[a, b]$, then

$$\int_a^b \left(\frac{\mathrm{d}}{\mathrm{d}x} F(x)\right) \mathrm{d}x = F(x)\Big|_a^b = F(b) - F(a)$$

**A remarkable statement** FTC Part 2 is usually stated as:

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$$

where $F(x)$ is any antiderivative of $f(x)$. Written like this, the formula can be interpreted as saying: *the area below a curve (by definition computed via a limit of a Riemann sum) can instead be computed by finding the antiderivative of the function which describes that curve and evaluating it at two points!*

**Limitations to FTC part 2** Some functions, such as $e^{-x^2}$ do not possess antiderivatives expressible in terms of elementary functions.[1] This means that to compute, say $\int_{-1}^{1} e^{-x^2}\,\mathrm{d}x$, one would need to resort to Riemann sums to get a numerical approximation. That is the best that we can do.[2]

**Generalized FTC Part 1**

The formula for FTC Part 1 can be generalized to

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_a^{g(x)} f(t)\,\mathrm{d}t = f(g(x)) \cdot g'(x)$$

and consequently

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{h(x)}^{g(x)} f(t)\,\mathrm{d}t = f(g(x)) \cdot g'(x) - f(h(x)) \cdot h'(x)$$

*Proof.* Let $F(x) = \displaystyle\int_a^x f(t)\,\mathrm{d}t$, so $F'(x) = f(x)$. Then

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\int_a^{g(x)} f(t)\,\mathrm{d}t &= \frac{\mathrm{d}}{\mathrm{d}x} F(g(x)) \\ &= F'(g(x)) \cdot g'(x) \\ &= f(g(x)) \cdot g'(x)\end{aligned}$$

which shows the first formula. The second formula follows from splitting the bounds of the integral and flipping one them, introducing a minus. □

[1] See https://www.wikiwand.com/en/Nonelementary_integral for more examples.

[2] This kind of computation is used in statistics, as this function gives the Gaussian (aka normal) distribution. They will probably just enter it into some calculator, but under the hood, this is what the calculator is doing.