

# How to compute sign charts quickly

2023 Fall

# The method

Given a rational function (polynomial over polynomial),

1. Put it into factored form.
2. Draw a number line and put tick marks for each zero and vertical asymptote (places where numerator=0 and where denominator = 0)
3. Going from right to left, determine the signature of each subinterval.

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## Why this is fast:

Crossing a tick mark will only affect one factor, flipping its sign. Whether that affects the sign of the entire function depends on what power the factor was raised to.

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## Why start from the right

Starting at the right will make all factors positive, unless you've written things oddly. One usually writes  $-(x - 3)$ , rather than  $(3 - x)$ . The sign of the rightmost interval is simply determined by the leading coefficients.

## Worked example

### Example

Determine the sign chart for the following function:

$$\frac{-4(x-3)^7(x-2)^8(x+3)^4}{x^6(x-4)^5(x+1)^3}$$

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The tick marks are  $x = 3, 2, -3, 0, 4, -1$ :

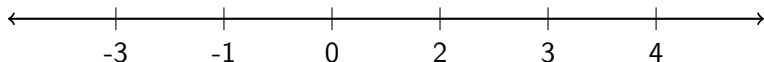
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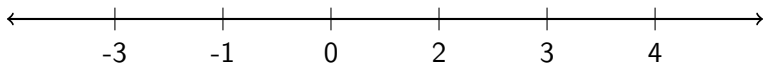


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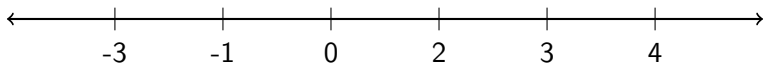


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Starting at the right, each factor is positive. The “−4” in the front makes the sign ‘−’:

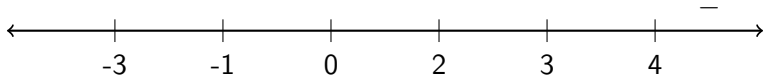
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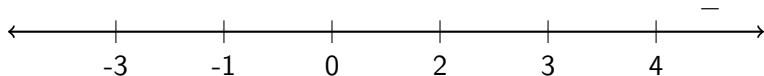
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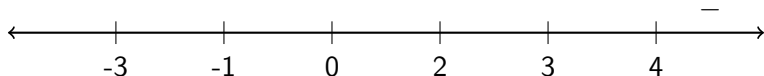


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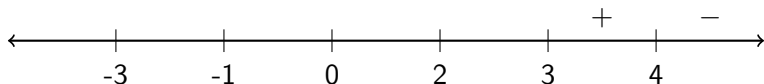
Moving to the left, we first cross  $x = 4$ , which corresponds to the factor  $(x - 4)$ . This flips the factor. It's power is 5, so the sign *does* flip on the number line.

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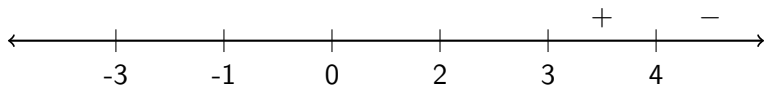
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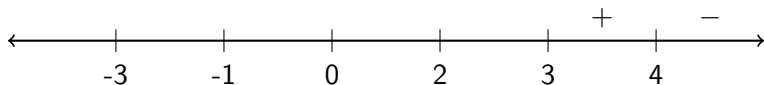


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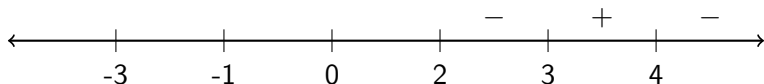
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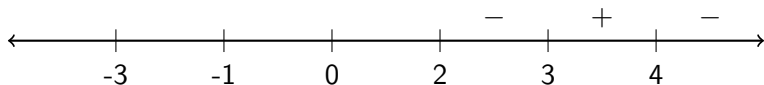


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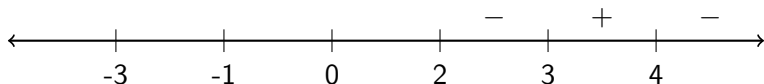


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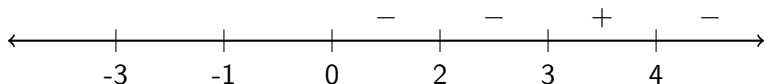
Next we cross  $x = 2$ . This time, the corresponding factor  $(x - 2)$  is raised to an even power. Although  $x - 2$  will flip in sign as we cross that tick mark, the factor  $(x - 2)^8$  will not, so the sign doesn't flip on the number line:

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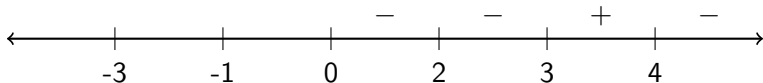
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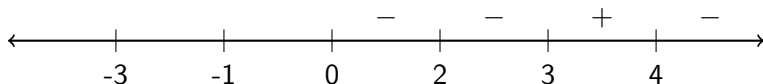
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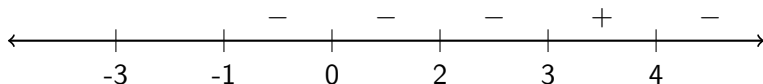
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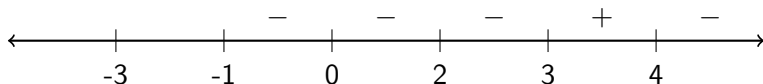
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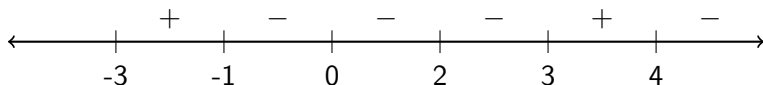
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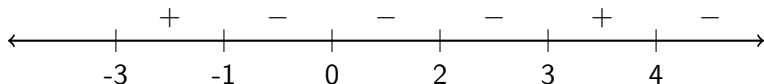


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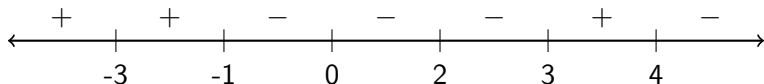
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