Problem

Compute the definite integral $\int_a^b f(x)\,\mathrm{d}x$, where the indefinite integral $\int f(x)\,\mathrm{d}x$ requires u-sub.

The two approaches

There are two slightly different strategies one can take. They are equivalent.

Approach 1: Convert bounds Perform the u-sub to convert the entire integral over

x world		u world
integrand in x	$\rightsquigarrow$	integrand in u
$\mathrm{d}x$	$\rightsquigarrow$	$\mathrm{d}u$
bounds in x	$\rightsquigarrow$	bounds in u

Take the antiderivative and evaluate in the u-world to get the answer. Schematically, this looks like:

$$\int_a^b f(x)\,\mathrm{d}x \overset{\text{u-sub}}{\rightsquigarrow} \int_n^m g(u)\,\mathrm{d}u = G(u)\Big|_n^m = G(m) - G(n)$$

Approach 2: Back-sub Perform the u-sub to convert the integral over *ignoring the bounds*

x world		u world
integrand in x	$\rightsquigarrow$	integrand in u
$\mathrm{d}x$	$\rightsquigarrow$	$\mathrm{d}u$

Take the antiderivative, back-substitute to get things in terms of x, then restore the bounds and evaluate in the x-world to get the answer. Schematically, this looks like:

$$\int_a^b f(x)\,\mathrm{d}x \overset{\text{u-sub}}{\rightsquigarrow} \int g(u)\,\mathrm{d}u = G(u) \overset{\text{back-sub}}{\rightsquigarrow} F(x)\Big|_a^b = F(b) - F(a)$$

Concrete example

Compute $\int_2^3 (1+x)^9 \, \mathrm{d}x$.

Approach 1: Convert bounds, evaluate in u

The u-sub we perform is

$$\begin{aligned} u &= 1+x \qquad & x = 2 \to u = 3 \\ \mathrm{d}u &= \mathrm{d}x \qquad & x = 3 \to u = 4 \end{aligned}$$

so

$$\int_2^3 (1+x)^9 \, \mathrm{d}x = \int_3^4 u^9 \, \mathrm{d}u = \frac{1}{10} u^{10} \Big|_3^4 = \boxed{\frac{1}{10}(4^{10} - 3^{10})}$$

Approach 2: Back-sub, evaluate in x

The u-sub we perform is

$$\begin{aligned} u &= 1+x \\ \mathrm{d}u &= \mathrm{d}x \end{aligned}$$

so

$$\int_2^3 (1+x)^9 \, \mathrm{d}x = \int u^9 \, \mathrm{d}u = \frac{1}{10} u^{10} = \frac{1}{10} (1+x)^{10} \Big|_2^3 = \boxed{\frac{1}{10}(4^{10} - 3^{10})}$$

Warning : A common mistake

Do not use x-bounds when evaluating in the u-world. You *will* get the wrong answer. Either convert the bounds over to the u-world, or back-substitute so that you can evaluate in the x-world.