NAME _____ Rec. Instructor: _____

Signature _____ Rec. Time _____

CALCULUS III - Exam 1

Show all work for full credit. No books or notes are permitted.

Problem	Points	Possible
1		20
2		20
3		30
4		20
5		10
Total Score		100

Note: Bold letters, like $\mathbf{u}$, are considered vectors unless specified otherwise.

You are free to use the following formulas on any of the problems.

Projection: $\text{proj}_{\mathbf{u}}\mathbf{v} = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|^2}\mathbf{u}$

Cylindrical Coordinates:

$$x = r\cos(\theta) \qquad r = \sqrt{x^2+y^2}$$
$$y = r\sin(\theta) \qquad \tan(\theta) = \frac{y}{x}$$
$$z = z \qquad z = z$$

Spherical Coordinates:

$$x = \rho\cos(\theta)\sin(\varphi) \qquad \rho = \sqrt{x^2+y^2+z^2}$$
$$y = \rho\sin(\theta)\sin(\varphi) \qquad \tan(\theta) = \frac{y}{x}$$
$$z = \rho\cos(\varphi) \qquad \cos(\varphi) = \frac{z}{\rho}$$

(20) **1.** Define $\mathbf{u} = \langle -1, 2, 5 \rangle$ and $\mathbf{v} = \langle 3, 2, 1 \rangle$. Compute the following.

a) $\|\mathbf{u}\|$.

b) $\mathbf{u} \cdot \mathbf{v}$.

c) $\mathbf{u} \times \mathbf{v}$.

d) The area of the triangle formed by $\mathbf{u}$ and $\mathbf{v}$.

e) The angle between $\mathbf{u}$ and $\mathbf{v}$.

(20) **2.** Let $\mathbf{u} = \langle 1, 2 \rangle$, and $\mathbf{v}$ be a vector of length 2 which is at an angle of $\frac{\pi}{3}$ to $\mathbf{u}$. Compute the following.

a) $\mathbf{u} \cdot \mathbf{v}$

b) $\|\mathbf{u} \times \mathbf{v}\|$

c) Assuming we let $\mathbf{w} = \mathbf{u} \times \mathbf{v}$, compute the following:

i) $\mathbf{u} \cdot \mathbf{w}$

ii) $\|\mathbf{u} \times \mathbf{w}\|$

(30) **3.**

a) Find an equation for the plane containing the points $P = (1, 0, 2)$, $Q = (-1, 3, 3)$, and $R = (0, -1, 1)$. Express your answer in the form $Ax + By + Cz = D$.

b) Find the shortest distance from point $S = (2, -1, -1)$ to the plane you found in (a).

c) Find the equation for the line passing through point R and perpendicular to the plane you found in (a).

(20) **4.** Convert the equation written in spherical coordinates into an equation in Cartesian coordinates.

$$\tan(\varphi)(\cos(\theta) - 2\sin(\theta)) = \rho$$

(10) **5.** Label the following as reasonable or unreasonable:

a) $\mathbf{u}/\|\mathbf{v}\|$

b) $\mathbf{u}/\mathbf{v}$

c) $(\mathbf{u} \cdot \mathbf{v}) \times \mathbf{w}$

d) $\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$

e) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$