NAME ______________________________ Rec. Instructor: __________________

Signature ______________________________ Rec. Time ____________

CALCULUS III - EXAM 2

<u>Show all work</u> for full credit. No books or notes are permitted.

Problem	Points	Possible
1		20
2		10
3		10
4		20
5		15
6		15
7		10
Total Score		100

Note: Bold letters, like $\mathbf{u}$, are considered vectors unless specified otherwise.

You are free to use the following formulas on any of the problems.

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$$

$$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$$

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3} = \frac{|y''|}{[1 + (y')^2]^{3/2}}$$

Directional Derivative: $D_{\mathbf{u}} f(x, y) = \nabla f(x, y) \cdot \mathbf{u}$.

(20) **1.** Find the unit tangent vector, the principal unit normal vector, the binormal vector, and curvature for

$$\mathbf{r}(t) = \langle \sqrt{2}t, e^t, e^{-t} \rangle.$$

(10) **2.** Let $f(x, y) = \ln(xy) - 1$, $y(u, v) = u^2 + v^2$ and $x(u, v) = uv$. Find $\dfrac{\partial f}{\partial u}$ and $\dfrac{\partial f}{\partial v}$.

(10) **3.** Calculate the following:

a) $\lim_{t \to 0} \left\langle \frac{e^t - 1}{t}, \frac{3}{5 - t^2}, \sin(2t) \right\rangle$.

b) The equation of the tangent plane to $f(x, y) = 2x^2 + xy - 6y^2$ at point $(3, -1)$.

(20) **4.** Find all the first partial derivatives and second partial derivatives of $f(x, y) = \sqrt{x^2 + y} + 3x$.

(15) **5.** Let

$$f(x, y, z) = xe^y + ye^z + ze^x.$$

a) Find the gradient of f.

b) Find $D_{\mathbf{u}}f(-1, 2, 0)$ in the direction of $\mathbf{v} = \langle -1, -1, -1 \rangle$.

(15) **6.** Calculate the limit if it exists. If the limit does not exist, explain why not.

a) $$\lim_{(x,y)\to(1,-1)} \frac{5x-7y}{x+y+1}$$

b) $$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$$

(10) **7.** Consider

$$\mathbf{r}(t) = \left\langle \frac{1}{3}t^3, \frac{\sqrt{2}}{2}t^2, t \right\rangle.$$

Find the arc length function $s(t)$ for $\mathbf{r}(t)$. (Hint: What is $(t^2+1)^2$?)