NAME ______________________ Rec. Instructor: ______________

Signature ______________________ Rec. Time ________

CALCULUS III - PRACTICE TEST 2

Show all work for full credit. No books or notes are permitted.

Problem	Points	Possible
1		
2		
3		
4		
5		
6		
7		
Total Score		

Note: Bold letters, like $\mathbf{u}$, are considered vectors unless specified otherwise.

You are free to use the following formulas on any of the problems.

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$$

$$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$$

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3} = \frac{|y''|}{[1+(y')^2]^{3/2}}$$

Directional Derivative: $D_{\mathbf{u}}f(x,y) = \nabla f(x,y) \cdot \mathbf{u}$.

1. Find the unit tangent vector, the principal unit normal vector, the binormal vector, and curvature for

$$\mathbf{r}(t) = \langle 3t, \cos(4t), \sin(4t) \rangle.$$

2. Let $f(x, y) = x^2 - xy + 3y^2$, $y(r, \theta) = r\sin(\theta)$ and $x(r, \theta) = r\cos(\theta)$. Find $\dfrac{\partial f}{\partial r}$ and $\dfrac{\partial f}{\partial \theta}$.

3. Calculate the following:

a) $\lim_{t\to\infty} \left\langle \frac{\ln(t)}{t^2}, \frac{2t^2}{1-t-t^2}, e^{-t} \right\rangle$.

b) The equation of the tangent plane to $f(x,y) = x^2y - \sqrt{x+y}$ at point $(1,2)$.

4. Find all the first partial derivatives and second partial derivatives of $f(x, y) = xy^2 \ln(x) + 3\cos(x)$.

5. Calculate the limit if it exists. If the limit does not exist, explain why not.

a) $\displaystyle \lim_{(x,y)\to(1,2)} \frac{-ye^x}{x+y^2}$

b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2}$

6. Let

$$f(x, y, z) = x^2y + y^2z + z^2x.$$

a) Find the gradient of f.

b) Find $D_{\mathbf{u}}f(1, 1, 1)$ in the direction of $\mathbf{v} = \langle\sqrt{2}, \sqrt{2}, \sqrt{2}\rangle$.

7. Consider $\mathbf{r}(t) = \langle 2t, 3\cos(2t), 3\sin(2t)\rangle$.

a) Find the arc length function $s(t)$ for $\mathbf{r}(t)$.

b) Find the arc length parameterization $\mathbf{r}(s)$.