NAME ______________________ Rec. Instructor: ______________

Signature ______________________ Rec. Time __________

CALCULUS III - PRACTICE TEST 3

Show all work for full credit. No books or notes are permitted.

Problem	Points	Possible
1		
2		
3		
4		
5		
6		
7		
Total Score		

Note: Bold letters, like **u**, are considered vectors unless specified otherwise.

You are free to use the following formulas on any of the problems.

Cylindrical Coordinates:

$$x = r\cos(\theta) \qquad r = \sqrt{x^2+y^2}$$

$$y = r\sin(\theta) \qquad \tan(\theta) = \frac{y}{x}$$

$$z = z \qquad z = z$$

$$dV = r dr d\theta dz$$

Spherical Coordinates:

$$x = \rho\cos(\theta)\sin(\varphi) \qquad \rho = \sqrt{x^2+y^2+z^2}$$

$$y = \rho\sin(\theta)\sin(\varphi) \qquad \tan(\theta) = \frac{y}{x}$$

$$z = \rho\cos(\varphi) \qquad \cos(\varphi) = \frac{z}{\rho}$$

$$dV = \rho^2\sin(\phi) d\rho d\theta d\phi$$

Second Derivative Test: Let $z = f(x, y)$ be a function of two variables for which the first- and second-order partial derivatives are continuous on some disk containing the point (x_0, y_0). Suppose $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$. Define the quantity

$$D = f_{xx}(x_0, y_0)f_{yy}(x_0, y_0) - (f_{xy}(x_0, y_0))^2$$

i. If $D > 0$ and $f_{xx}(x_0, y_0) > 0$, then f has a local minimum at (x_0, y_0).
ii. If $D > 0$ and $f_{xx}(x_0, y_0) < 0$, then f has a local maximum at (x_0, y_0).
iii. If $D < 0$, then f has a saddle point at (x_0, y_0).
iv. If $D = 0$, then the test is inconclusive.

Change of Variables: We have $T : S \to R$ and $T(u, v) = (g(u, v), h(u, v))$.
$\text{Jac}(T) = \left|\dfrac{\partial(x, y)}{\partial(u, v)}\right|$ and

$$\int\int_S f(x, y)dxdy = \int\int_S f(g(u, v), h(u, v))|\text{Jac}(T)|dudv.$$

1. Integrate $f(x, y) = x$ over the region bounded by $y = x^2$ and $y = x + 2$.

2. Let R be the region given by $x \geq 0$, $y \leq 0$, and $x^2 + y^2 \leq z \leq 4$.

a) If you were integrating over R, would you integrate the region in Cartesian, Polar, Cylindrical, or Spherical Coordinates? Explain why you choose the coordinate system you did, and express the bounds for the region R in that system.

b) Find the volume of R.

3. Find and classify all the critical points of $3x^2y + y^3 - 3x^2 - 3y^2 + 2$.

4. Use Lagrange Multipliers to find the maximum and minimum of the following function

$$f(x, y, z) = x + y^2 - z$$

subject to the constraint

$$g(x, y, z) = x^2 + y^2 + z^2 = 1.$$

5. Let E be the region inside $x^2 + y^2 + z^2 = 25$ and $z = -\sqrt{3x^2 + 3y^2}$, and where $y \geq 0$.

a) If you were integrating over E, would you integrate the region in Cartesian, Polar, Cylindrical, or Spherical Coordinates? Explain why you choose the coordinate system you did, and express the bounds for the region E in that system.

b) Evaluate

$$\int\int\int_E x dV.$$

6. Evaluate

$$\int\int_R x + y dA$$

where R is the trapezoidal region with vertices given by the points $(0, 0)$, $(2, 0)$, $(1, 1)$ and $(1, -1)$ using the transformation $x(u, v) = 2u + 3v$ and $y(u, v) = 2u - 3v$.

7. Answer the following short answer questions.

a) Give an example of a region that is Type 2, but not Type 1. Write out the region explicitly, don't just graph it.

b) Write out the Extreme Value Theorem (Theorem 4.18 from the textbook).

c) If the area of a region E is 2, and $\int\int\int_E f(x,y,z)dV = 3$, what is the average value of $f(x,y,z)$?.

d) If I set up an integral that looks like:

$$\int_0^1 \int_0^z \int_0^{x+y} f(x,y,z)dzdydx$$

what is the problem with my integral set up?

e) Suppose we want to integrate

$$\int\int_R y\sin(xy)dA$$

over $R = [1,2] \times [0,\pi]$. Which is less steps, integrating with respect to x first, or y first? Why?