Contents

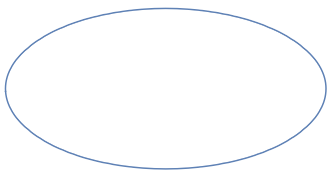
1 Conic Sections

A conic in general form can be written as

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

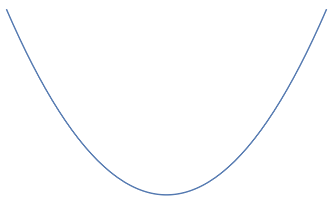
Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



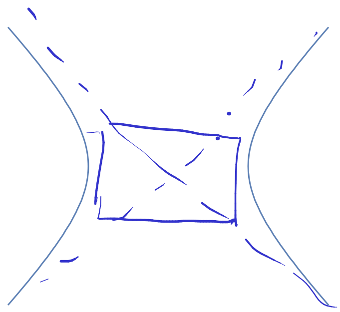
Parabola

$$y = ax^2$$



Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



2 Quadric surfaces

Quadric surfaces are the graphs of equations that can be expressed in the form

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Jz + K = 0.$$

Ellipsoid

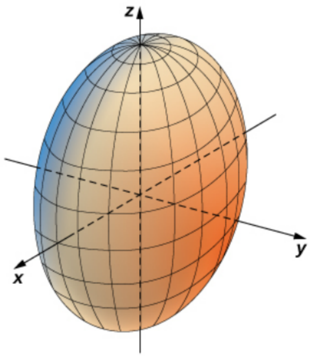
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

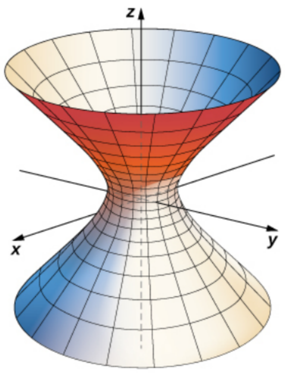
Hyperboloid (one sheet)

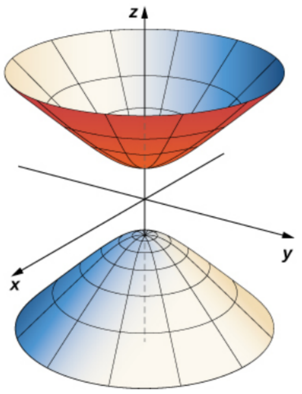
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Hyperboloid (two sheets)

$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$







Elliptic paraboloid

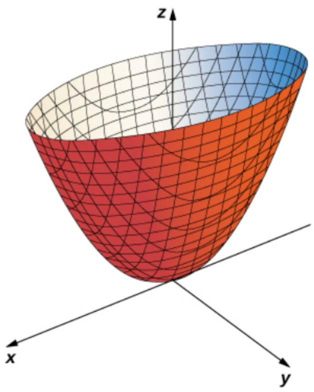
$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

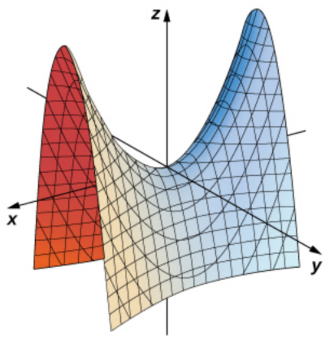
Hyperbolic paraboloid

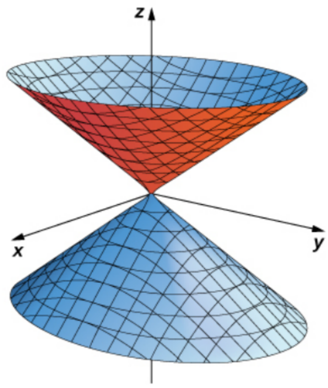
$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

Elliptic Cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$







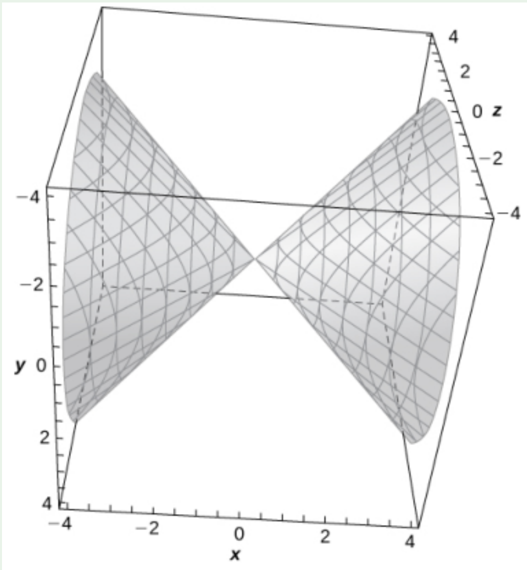
3 Visualizing things in 3-space

Definition. The **traces** of a surface are the cross-sections created from intersecting the surface with a plane parallel to one of the coordiante planes.

Software `https://c3d.libretexts.org/CalcPlot3D/index.html`
Use "Implicit Surface" to draw these.

4 Example problems

Example (Similar to Exercise 311)**.** A graph of a quadric surface is given.



(a) Specify the name of the quadric surface.

(b) Determine the axis of symmetry of the quadric surface.

Answer. Elliptic cone. x-axis of symmetry.

Example (Similar to Exercise 319)**.** Rewrite the given equation of the quadric surface in standard form. Identify the surface.

$$-3x^2 + 5y^2 - z^2 = 10$$

Solution. Dividing to get into standard form,

$$-\frac{x^2}{\frac{10}{3}} + \frac{y^2}{2} - \frac{z^2}{10} = 1$$

This is a $\boxed{\text{two-sheeted hyperboloid with the } y\text{-axis of symmetry}}$

Example (Similar to Exercise 340)**.** The equation of a quadric surface is given.

(a) Use the method of completing the square to write the equation in stnadard form.

(b) Identify the surface

$$x^2 + 4y^2 - 4z^2 - 6x - 16y - 16z + 5 = 0$$

$x^2 + 2a + a^2 = (x + a)^2$

Solution.

$$(x^2 - 6x + 9) + 4(y^2 - 4y + 4) - 4(z^2 + 4z + 4) + 5 = 9 + 16 - 16$$
$$(x - 3)^2 \cancel{2} + 4(y - 2)^2 - 4(z + 2)^2 = 4$$
$$\boxed{\frac{(x - 3)^2}{4} + (y - 2)^2 - (z + 2)^2 = 1}$$

This is a $\boxed{\text{one-sheeted hyperboloid centered at } (3, 2, -2) \text{ with } z\text{-axis of symmetry}}$