Contents

1 Cylindrical Coordinates

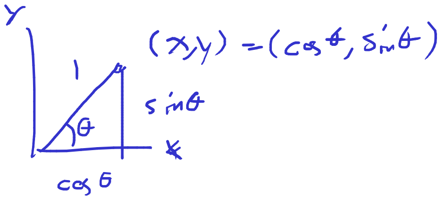
Cylindrical is just polar + z axis. (r, θ, z)

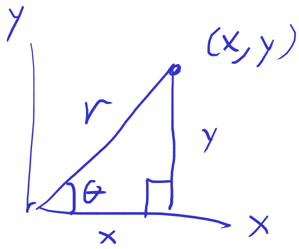
Conversion formulas

$$x = r\cos\theta \qquad r^2 = x^2 + y^2$$

$$y = r\sin\theta \qquad \tan\theta = \frac{y}{x}$$

$$z = z \qquad z = z$$





$x^2 + y^2 = r^2$

$\tan\theta = \frac{y}{x}$

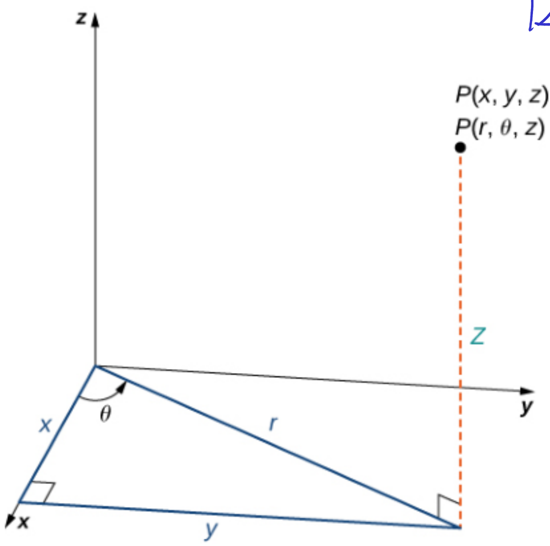


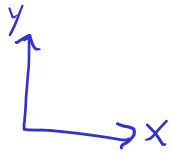
Figure 2.89 The right triangle lies in the *xy*-plane. The length of the hypotenuse is r and θ is the measure of the angle formed by the positive *x*-axis and the hypotenuse. The *z*-coordinate describes the location of the point above or below the *xy*-plane.

2 Spherical Coordinates

(ρ, θ, ϕ)

- ρ: Radius. Distance to origin
- θ: Angle in xy-plane. Like polar. Should be within $[0, 2\pi)$
- ϕ: Angle of declination from the positive z-axis. Should be within $[0, \pi]$

φ



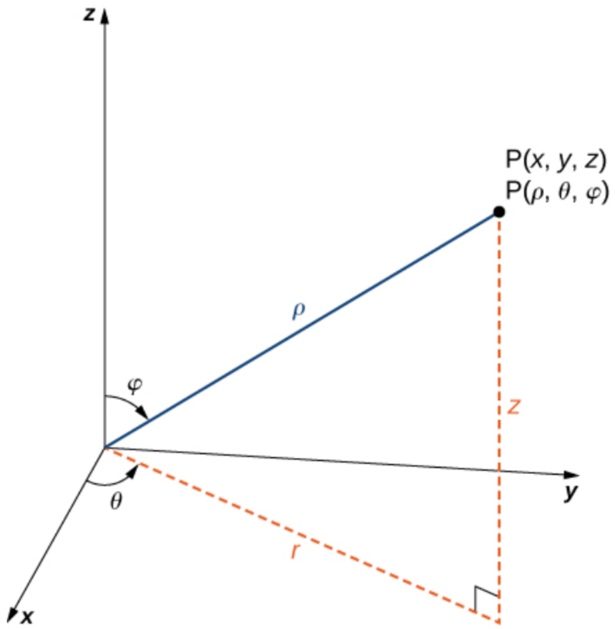


Figure 2.97 The relationship among spherical, rectangular, and cylindrical coordinates.

Converting Between Spherical and Rectangular Coordinates

$$
\begin{aligned}
x &= \rho \sin\varphi \cos\theta & \rho^2 &= x^2 + y^2 + z^2 \\
y &= \rho \sin\varphi \sin\theta & \tan\theta &= \frac{y}{x} \\
z &= \rho \cos\varphi & \varphi &= \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right).
\end{aligned}
$$

Converting between Cylindrical and Spherical Directly

$$
\begin{aligned}
r &= \rho \sin\varphi & \rho &= \sqrt{r^2 + z^2} \\
\theta &= \theta & \theta &= \theta \\
z &= \rho \cos\varphi & \varphi &= \arccos\left(\frac{z}{\sqrt{r^2+z^2}}\right)
\end{aligned}
$$

3 Example Problems

Example (Similar to Exercise 363)**.** A point $(r, \theta, z) = (3, \frac{\pi}{3}, 5)$ is given in cylindrical coordinates. Find the rectangular coordinates of the point.

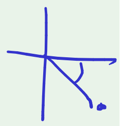
Solution.

$$\begin{aligned} x &= 3\cos\frac{\pi}{3} = \frac{3}{2} \\ y &= 3\sin\frac{\pi}{3} = \frac{3\sqrt{3}}{2} \\ z &= 5 \\ (x, y, z) &= \boxed{\left(\tfrac{3}{2}, \tfrac{3\sqrt{3}}{2}, 5\right)} \end{aligned}$$

Example (Similar to Exercise 370)**.** A point $(x, y, z) = (3, -3, 7)$ is given in rectangular coordinates. Find the cylindrical coordinates of the point.

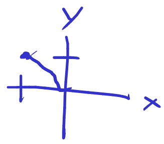
Solution.

$$\begin{aligned} r^2 = x^2 + y^2 = 9 + 9 = 18 &\implies r = 3\sqrt{2} \\ \tan\theta = \frac{-3}{3} = -1 &\implies \theta = -\frac{\pi}{4} \\ z = 7 & \end{aligned}$$



so the coordinate is $(r, \theta, z) = \boxed{(3\sqrt{3}, -\frac{\pi}{4}, 7)}$

$\theta = \frac{7\pi}{4}$



Remark. Should check that the θ computed from taking inverse tangent lies in the correct quadrant! The point $(-3, 3, 7)$ would also yield $\theta = \tan^{-1}(-1) = -\frac{\pi}{4}$, but this point should lie over quadrant II, hence the correct angle should be $\theta = \frac{3\pi}{4}$.

Example (Similar to Exercise 380)**.** The equation of a surface in rectangular coordinates is given. Find the equation of the surface in cylindrical coordinates.
$y = 2$

Solution. Using the conversion formulas, we get

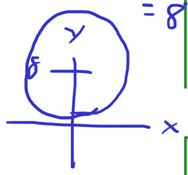
$$r\sin\theta = 2 \implies \boxed{r = 2\csc\theta}$$

Example (Similar to Exercise 382). The equation of a surface in rectangular coordinates is given. Find the equation of the surface in cylindrical coordinates.
$x^2 + y^2 - 16x = 0$

$x^2+y^2-16x+64=64$

$x^2+(y-8)^2=64$

$=8^2$



Solution. Using the conversion formulas, we get

$$r^2 - 16r\cos\theta = 0 \implies r(r - 16\cos\theta) = 0$$
$$\implies \boxed{r = 16\cos\theta}$$

Example (Similar to Exercise 394). The equation of a surface in spherical coordinates is given. Find the equation of the surface in rectangular coordinates. Identify and graph the surface.
$\rho = 2\cos\varphi$

Solution.

$$\Rightarrow \rho^2 = 2\rho\cos\varphi$$
$$\Rightarrow x^2 + y^2 + z^2 = 2z$$
$$\Rightarrow x^2 + y^2 + z^2 - 2z + 1 = 1$$
$$\Rightarrow x^2 + y^2 + (z-1)^2 = 1$$

$\boxed{\text{Sphere of radius 1 centered at } (0,0,1)}$