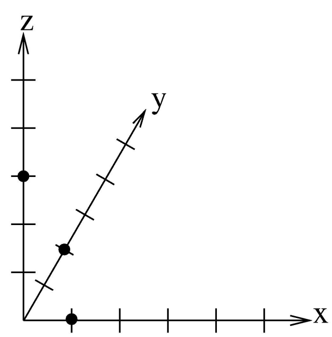
Selected Answers to Practice Midterm # 1 (The actual midterm will be shorter!)

Math 220, Fall 2020

1. Plot the points $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$ in a three-dimensional coordinate system. Label the axes. (Note: You should label everything in a way that makes the right hand rule correct!)



2. Which of the following are reasonable expressions? (**RE**) Which are not? (**NR**)

$(\vec{u} \times \vec{v}) \bullet \vec{w}$	**RE**
$\vec{w} \bullet (\vec{u} \times \vec{v})$	**RE**
$\vec{u} \times (\vec{v} \bullet \vec{w})$	**NR**
$(\vec{v} \bullet \vec{w}) \times \vec{u}$	**NR**
$(\vec{u} \times \vec{v}) \bullet (\vec{w} \bullet \vec{x})$	**NR**
$\frac{\vec{u}}{\vec{w}}$	**NR**
$\frac{\vec{u}}{\lvert\vec{w}\rvert}$	**RE**
$3\vec{u} - 4\vec{w}$	**RE**
$\lvert\vec{u} - 2\vec{w}\rvert + \lvert\vec{u} + 2\vec{w}\rvert$	**RE**
$\lvert 3 - \vec{u}\rvert$	**NR**

The next few problems all refer to the following pairs of vectors:

(a) $\vec{u} := (3, 4)$ and $\vec{w} := (5, -1)$.
(b) $\vec{u} := (2, 5, -3)$ and $\vec{w} := (4, -1, 6)$.
(c) $\vec{u} := 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{w} := 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$.

3. Find the norm of every vector above.

4. Find the distance from $\vec{u}$ to $\vec{w}$ for each pair of vectors above.

5. Find the dot products $\vec{u} \bullet \vec{w}$ and $\vec{w} \bullet \vec{u}$ and the cross products $\vec{u} \times \vec{w}$ and $\vec{w} \times \vec{u}$ for each pair of vectors above.

6. Find the area of the parallelogram spanned by $\vec{u}$ and $\vec{w}$ for each pair of vectors above.

7. Use dot products to write an exact formula for the angle between $\vec{u}$ and $\vec{w}$ for each pair above.

8. Use cross products to write an exact formula for the angle between $\vec{u}$ and $\vec{w}$ for each pair above. (Note: It is NOT acceptable to need a calculator to know things like $\sin 0 = 0$ and $\cos 0 = 1$. You should be able to make a decent sketch of the sine and cosine functions.)

OK, so I will do everything all at once for each pair of vectors for the last six problems. I will use θ to denote the angle between $\vec{u}$ and $\vec{w}$. (Also, θ can always be chosen to be in the interval $[0, \pi]$.) **Immediate disclaimer:** Although I spent a long time answering question # 8, please do **not** take this to mean that I view it as more important than other similar problems. In fact the opposite is true! It is one of the least important problems on this practice test. I guess when I made this test I wanted to make sure that there was something which tested the important identity:

$$|\vec{u} \times \vec{w}| = |\vec{u}| \cdot |\vec{w}| \cdot \sin\theta \ .$$

That identity by itself is quite important. The trigonometric acrobatics involved in giving a really good answer to # 8 are totally unimportant.

If you do not understand that stuff, then don't panic. Anyway ... Onwards!

(a) $\vec{u} := (3,4)$ and $\vec{w} := (5,-1)$.

$|\vec{u}| = \sqrt{3^2+4^2} = 5$. $|\vec{w}| = \sqrt{5^2+(-1)^2} = \sqrt{26}$.

$|\vec{u}-\vec{w}| = |(3-5,\ 4-(-1)| = \sqrt{2^2+5^2} = \sqrt{29}$.

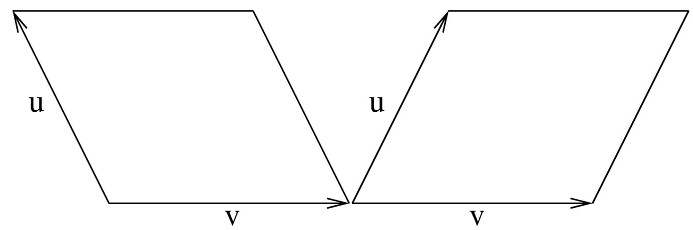
$\vec{u} \bullet \vec{w} = 3 \cdot 5 + 4 \cdot (-1) = 11$. $\vec{w} \bullet \vec{u} = 11$.

In order to do cross products, you must interpret the vector $(3,4)$ as being embedded in three dimensional space, or, in other words, view it as $(3,4,0)$, and similarly $(5,-1)$ is viewed as $(5,-1,0)$. Then we have
$\vec{u} \times \vec{w} = (0,0,-23)$. $\vec{w} \times \vec{u} = (0,0,23)$.

Area of parallelogram is the norm of the cross product which is 23. On the other hand, if we had not just computed the cross product, then we could compute this area faster by doing the two by two determinant of the matrix whose column vectors are given by $\vec{u}$ and $\vec{w}$.

$$\theta = \arccos\left(\frac{\vec{u} \bullet \vec{w}}{|\vec{u}| \cdot |\vec{w}|}\right) = \arccos\left(\frac{11}{5\sqrt{26}}\right) .$$

Regarding the exact formula for the angle using cross products, there is actually a rather troublesome issue. I would go so far as to describe the issue as dicey. Perhaps even prickly. The problem can be understood in a few ways. One way to understand it is to observe the following picture:



Notice that although the angle between $\vec{u}$ and $\vec{v}$ is obtuse on the left and acute on the right, the area of the parallelograms is the same. So, knowing the length of $\vec{u}$ and $\vec{v}$ and knowing the area of the parallelogram that they span cannot allow you to determine the angle between $\vec{u}$ and $\vec{v}$. (It allows you to narrow it down to two possibilities.)

Another way to discover the same problem is to observe that if $0 \leq \theta \leq \pi/2$, then $\sin\theta = \sin(\pi - \theta)$. The graph of the sine function is symmetric about the axis $\theta = \pi/2$. This problem is exactly like the problem of finding what x is, if you know that $x^2 = 16$. If you take the square root of 16 your calculator will give you the number 4. (The square root function is **defined** to be positive, so the solution to the equation $x^2 = 3$ is **not** $x = \sqrt{3}$, but it **is** $x = \pm\sqrt{3}$.) Similarly, if you take the arcsine of a number between 0 and 1, then you get an angle between 0 and $\pi/2$. On the other hand, the angle between $\vec{u}$ and $\vec{w}$ should be a number between 0 and π. In order to properly invert the sine function we need to first know whether the angle is between 0 and $\pi/2$, or between $\pi/2$ and π. The most reasonable way of doing this is by taking the dot product. To summarize and especially to deal with the case where the dot product is negative we have the following procedure:

$$\text{If } \vec{u} \bullet \vec{w} \ \geq 0 \text{ then } \theta = \arcsin\left(\frac{|\vec{u} \times \vec{w}|}{|\vec{u}| \cdot |\vec{w}|}\right)$$

$$\text{If } \vec{u} \bullet \vec{w} \ < 0 \text{ then } \theta = \pi - \arcsin\left(\frac{|\vec{u} \times \vec{w}|}{|\vec{u}| \cdot |\vec{w}|}\right)$$

Having said all of this stuff, **what I most want you to see is that there is never any problem like this one when using**

the angle formula found from the dot product, so it is a better way to find the angle. Stated more succinctly, if you don't follow everything above, then don't lose too much sleep over it. It is not going to "make or break you" on the test...

So, since $\vec{u} \times \vec{w} > 0$, we have:

$$\theta = \arcsin\left(\frac{|\vec{u} \times \vec{w}|}{|\vec{u}| \cdot |\vec{w}|}\right) = \arcsin\left(\frac{23}{5\sqrt{26}}\right) .$$

(b) $\vec{u} := (2, 5, -3)$ and $\vec{w} := (4, -1, 6)$.

$|\vec{u}| = \sqrt{2^2 + 5^2 + (-3)^2} = \sqrt{38}$. $|\vec{w}| = \sqrt{53}$.

$|\vec{u} - \vec{w}| = |(-2, 6, -9)| = \sqrt{121} = 11$.

$\vec{u} \bullet \vec{w} = 8 - 5 - 18 = -15$. $\vec{w} \bullet \vec{u} = -15$.

$\vec{u} \times \vec{w} = (27, -24, -22)$. $\vec{w} \times \vec{u} = (-27, 24, 22)$.

Area of parallelogram is the norm of the cross product which is $\sqrt{27^2 + (-24)^2 + (-22)^2} = \sqrt{1789}$. Notice that we can't just do a two by two determinant this time.

$$\theta = \arccos\left(\frac{\vec{u} \bullet \vec{w}}{|\vec{u}| \cdot |\vec{w}|}\right) = \arccos\left(\frac{-15}{\sqrt{38 \cdot 53}}\right) = \arccos\left(\frac{-15}{\sqrt{2014}}\right) .$$

For the cross product part, notice first that $\vec{u} \bullet \vec{w} < 0$. Then by the discussion above we have

$$\theta = \pi - \arcsin\left(\frac{|\vec{u} \times \vec{w}|}{|\vec{u}| \cdot |\vec{w}|}\right) = \pi - \arcsin\left(\frac{\sqrt{1789}}{\sqrt{2014}}\right) = \pi - \arcsin\left(\sqrt{\frac{1789}{2014}}\right) .$$

(c) $\vec{u} := 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{w} := 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. and this is the same as: $\vec{u} := (3, 2, -1)$ and $\vec{w} = (4, -3, 2)$.

$|\vec{u}| = \sqrt{14}$. $|\vec{w}| = \sqrt{29}$.

$|\vec{u} - \vec{w}| = |(-1, 5, -3)| = \sqrt{35}$.

$\vec{u} \bullet \vec{w} = 12 - 6 - 2 = 4$. $\vec{w} \bullet \vec{u} = 4$.

$\vec{u} \times \vec{w} = (1, -10, -17)$. $\vec{w} \times \vec{u} = (-1, 10, 17)$.

Area of parallelogram is the norm of the cross product which is $\sqrt{1^2 + (-10)^2 + (-17)^2} = \sqrt{390}$. Again, we cannot do a two by two determinant.

$$\theta = \arccos\left(\frac{\vec{u} \bullet \vec{w}}{|\vec{u}| \cdot |\vec{w}|}\right) = \arccos\left(\frac{4}{\sqrt{14 \cdot 29}}\right) = \arccos\left(\frac{4}{\sqrt{406}}\right) .$$

For the cross product part, notice first that $\vec{u} \bullet \vec{w} \geq 0$ again. Thus we can compute simply:

$$\theta = \arcsin\left(\frac{|\vec{u} \times \vec{w}|}{|\vec{u}| \cdot |\vec{w}|}\right) = \arcsin\left(\frac{\sqrt{390}}{\sqrt{406}}\right) = \arcsin\left(\sqrt{\frac{195}{203}}\right) .$$

9. Find the volume of the parallelpiped spanned by:

(a) $\vec{a} := (1, 2, 3)$, $\vec{b} := (1, 3, 5)$, and $\vec{c} := (4, 1, -2)$.

The volume is the absolute value of the determinant whose column vectors (or row vectors) are given by the vectors above. So, we compute:

$$\begin{vmatrix} 1 & 1 & 4 \\ 2 & 3 & 1 \\ 3 & 5 & -2 \end{vmatrix} = (4\cdot 2\cdot 5 + 1\cdot 3\cdot(-2) + 1\cdot 1\cdot 3)$$

$$\begin{aligned} &- (1\cdot 2\cdot(-2) + 4\cdot 3\cdot 3 + 1\cdot 1\cdot 5) \\ &= (40 - 6 + 3) - (-4 + 36 + 5) \\ &= (37) - (37) \\ &= 0\ . \end{aligned}$$

Ugh. I guess the three vectors all lie within the same plane so they don't really span a parallelpiped.

(b) $\vec{a} := \hat{\mathbf{i}} - \hat{\mathbf{k}}$, $\vec{b} := \hat{\mathbf{j}} + \hat{\mathbf{k}}$, and $\vec{c} := \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$.

Same computation ... new vectors:

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = (0 + 1 + 0) - (0 - 1 + 1)$$

$$= 1\ .$$

10. Give the parametric equation and the symmetric equations for the line passing through the points $(2, 3, 4)$ and $(-2, -5, 1)$.

First note that there are multiple correct answers. Here is one:

$$x(t) = 2 + (2 - (-2))t, \quad y(t) = 3 + (3 - (-5))t, \quad z(t) = 4 + (4 - 1)t\ .$$

Obviously one should simplify these expressions and get:

$$x(t) = 2 + 4t, \quad y(t) = 3 + 8t, \quad z(t) = 4 + 3t\ .$$

The symmetric equations for this line are:

$$\frac{x-2}{4} = \frac{y-3}{8} = \frac{z-4}{3}\ .$$

11. Give an expression for the plane containing the line from the previous problem and the point $(1, -1, 2)$.

    There are many ways to express the answer. Here's one:

$$(x-2, y-3, z-4) \bullet (4, -5, 8) = 0 \ .$$

12. Give an expression for the plane which contains the point $(2, 3, 4)$ and which is perpendicular to the vector $(-1, -4, 7)$.

    Again, there are many ways to express the answer, although in this problem, if your answer isn't the following one, then you probably did more work than you needed to...

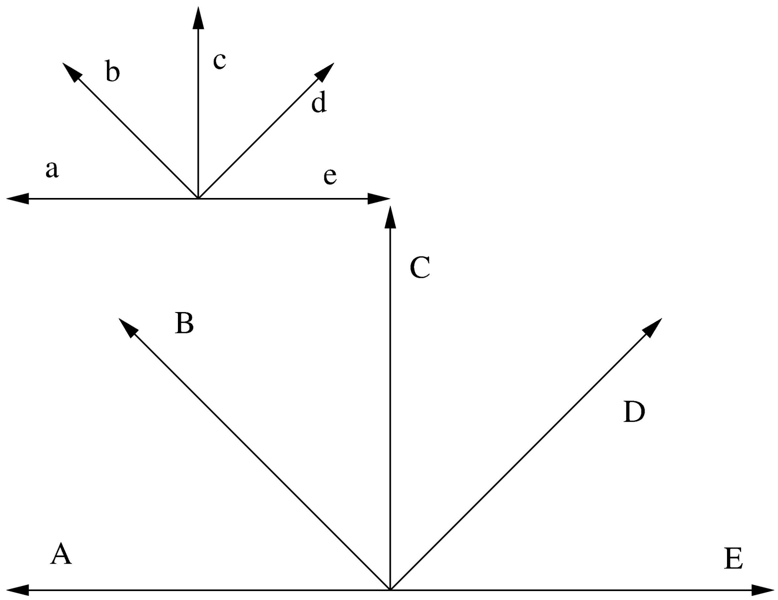
$$(x-2, y-3, z-4) \bullet (-1, -4, 7) = 0 \ .$$

13. Find an expression for the plane which contains the points $(5, 6, 7)$, $(1, 2, -1)$, and $(3, -1, 2)$.

$$(x-5, y-6, z-7) \bullet (9, 1, -5) = 0 \ .$$

14. Give an expression for the sphere with center $(2, -4, -7)$ and radius equal to 5.

$$(x-2)^2 + (y+4)^2 + (z+7)^2 = 5^2 \ .$$



For the next few problems we refer to the figure above which is drawn to scale. The vectors a, b, c, d, and e all have unit length, and the vectors A, B, C, D, and E all have length two. All of the angles between the vectors are multiples of 45 degrees.

15. Put the numbers in order from smallest to largest:

(a) $|A|$, $|a|$, $|A-E|$, $|C-E|$, and $|d-e|$.

$$|d-e| < 1 = |a| < 2 = |A| < 2\sqrt{2} = |C-E| < 4 = |A-E|.$$

(b) $a \bullet e$, $a \bullet d$, $a \bullet c$, $a \bullet b$, and $a \bullet a$.

$$-1 = a \bullet e < -\tfrac{\sqrt{2}}{2} = a \bullet d < 0 = a \bullet c < \tfrac{\sqrt{2}}{2} = a \bullet b < 1 = a \bullet a.$$

(c) $|a \times e|$, $|d \times a|$, $|a \times c|$, and $|A \times C|$.

$$0 = |a \times e| < \tfrac{\sqrt{2}}{2} = |d \times a| < 1 = |a \times c| < 4 = |A \times C|.$$

16. Assuming that you are viewing this page normally, which of the following vectors is pointing toward you? (Of course I should have said something like "assuming that you are viewing the figure above normally.")

    (a) $A \times B$ is pointing away from me.

    (b) $D \times C$ is pointing toward me.

    (c) $A \times D$ is pointing away from me.

    (d) $E \times B$ is pointing toward me.

17. Compute the following explicitly:

    (a)
    $$E \bullet A = -4.$$

    (b)
    $$B \bullet B = 4.$$

    (c)
    $$|a \times c| = 1.$$

    (d)
    $$|A \times C| = 4.$$

    (e)
    $$|a \times a| = 0.$$

    (f)
    $$C \bullet A = 0.$$

    (g)
    $$\frac{D \bullet E}{d \bullet e} = 4\ .$$

    (h)
    $$\frac{C \bullet B}{C \bullet b} = 2\ .$$

18. Find the shortest distance from the point $(3, 1, -2)$ to the plane given by $2x - 3y + z = 5$.

We can find this distance by finding the scalar projection of any vector which goes from the plane to the point $(3, 1, -2)$ onto the vector normal to the plane. So, first we need a point in the plane. $(1, -1, 0)$ is the first one I saw, although $(0, 0, 5)$ is probably even easier. If you take the first one, then you might observe that the plane can be written

$$2(x-1) - 3(y+1) + z = 0 \quad \text{or} \quad (2, -3, 1) \bullet (x-1, y+1, z) = 0 \; .$$

If you take the second one, then you might observe that the plane can be written

$$2x - 3y + 1(z-5) = 0 \quad \text{or} \quad (2, -3, 1) \bullet (x, y, z-5) = 0 \; .$$

Either way, $\vec{N} = (2, -3, 1)$ is a normal vector to the plane. Now you need a vector from the point you found to $(3, 1, -2)$. In the first case, you would most naturally get $(3-1, 1-(-1), -2-0) = (2, 2, -2)$ and in the second case you'd most naturally get $(3, 1, -7)$. Time to compute the scalar projection. First case:

$$(2, 2, -2) \bullet \frac{\vec{N}}{|\vec{N}|} = \frac{(2, 2, -2) \bullet (2, -3, 1)}{\sqrt{4+9+1}} = \frac{4-6-2}{\sqrt{14}} = \frac{-4}{\sqrt{14}}$$

so the distance you'd get is $\frac{4}{\sqrt{14}}$. Second case:

$$(3, 1, -7) \bullet \frac{\vec{N}}{|\vec{N}|} = \frac{(3, 1, -7) \bullet (2, -3, 1)}{\sqrt{4+9+1}} = \frac{6-3-7}{\sqrt{14}} = \frac{-4}{\sqrt{14}}$$

so you'd get the same thing, which should make you happy and joyous.

19. Hope you did this!

20. Hope you did this as well!

21. Make the following definitions

    (a) $f(t) := e^{2t}, \; g(t) := \sin(3t)$.

(b) $\vec{u}(t) := (5t,\ e^{-3t},\ \cos(7t)),\ \vec{w}(t) := (e^{t},\ \sin(4t),\ t^{8})$.

$$\begin{aligned}\frac{d}{dt}\left(f(t)\vec{u}(t)+g(t)\vec{w}(t)\right) &= f'(t)\vec{u}(t)+f(t)\vec{u}'(t)+g'(t)\vec{w}(t)+g(t)\vec{w}'(t)\\ &= 2e^{2t}(5t,\ e^{-3t},\ \cos(7t))\\ &\quad + e^{2t}(5,\ -3e^{-3t},\ -7\sin(7t))\\ &\quad + 3\cos(3t)(e^{t},\ \sin(4t),\ t^{8})\\ &\quad + \sin(3t)(e^{t},\ 4\cos(4t),\ 8t^{7})\ ,\end{aligned}$$

and now you should combine each of the corresponding components. (Of course your answer is a **vector!**)

$$\begin{aligned}\frac{d}{dt}\left(\vec{u}(t)\bullet\vec{w}(t)\right) &= \vec{u}'(t)\bullet\vec{w}(t)+\vec{u}(t)\bullet\vec{w}'(t)\\ &= 5e^{t}-3e^{-3t}\sin(4t)-7t^{8}\sin(7t)\\ &\qquad + 5te^{t}+4e^{-3t}\cos(4t)+8t^{7}\cos(7t)\ .\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\left(\vec{u}(t)\times\vec{w}(t)\right) &= \vec{u}'(t)\times\vec{w}(t)+\vec{u}(t)\times\vec{w}'(t)\\ &= (5,\ -3e^{-3t},\ -7\sin(7t))\times(e^{t},\ \sin(4t),\ t^{8})\\ &\quad + (5t,\ e^{-3t},\ \cos(7t))\times(e^{t},\ 4\cos(4t),\ 8t^{7})\\ &= (7\sin(4t)\sin(7t)-3e^{-3t}t^{8}, -7\sin(7t)e^{t}-5t^{8}, 5\sin(4t)+3e^{-2t})\\ &\quad + (8t^{7}e^{-3t}-4\cos(4t)\cos(7t), e^{t}\cos(7t)-40t^{8}, 20t\cos(4t)-e^{-2t})\ ,\end{aligned}$$

and now you should combine each of the corresponding components again.

$$\begin{aligned}\frac{d}{dt}\left(\vec{u}(f(t))\right) &= f'(t)\vec{u}'(f(t))\\ &= 2e^{2t}(5,\ -3e^{-3e^{2t}},\ -7\sin(7e^{2t}))\end{aligned}$$

22. ~~See the link to happiness and joy!!!~~

Typeset from https://www.math.ksu.edu/~blanki/HappyJoy1.jpg and https://www.math.ksu.edu/~blanki/HappyJoy2.jpg

#22 (a)

$$\begin{aligned}\mathbf{r}(t) &= \left\langle \sin(2t), \cos(2t), 7 \right\rangle \\ \mathbf{r}'(t) &= \left\langle 2\cos(2t), -2\sin(2t), 0 \right\rangle \\ \mathbf{r}''(t) &= \left\langle -4\sin(2t), -4\cos(2t), 0 \right\rangle\end{aligned}$$

Unit Tangent Vector

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\left\|\mathbf{r}'(t)\right\|} = \frac{(2\cos(2t), -2\sin(2t), 0)}{\sqrt{4\cos^2(2t) + 4\sin^2(2t)}} = \boxed{\left\langle \cos(2t), -\sin(2t), 0 \right\rangle}$$

Unit normal

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\left\|\mathbf{T}'(t)\right\|} = \frac{\left\langle -2\sin(2t) - 2\cos(2t), 0 \right\rangle}{\sqrt{4\sin^2(2t) + 4\cos 2(2t)}} = \boxed{\left\langle -\sin(2t), -\cos(2t), 0 \right\rangle}$$

Unit binormal

$$\mathbf{B}(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos(2t) & -\sin(2t) & 0 \\ -\sin(2t) & -\cos(2t) & 0 \end{vmatrix} = \left(-\cos^2(2t) - \sin^2(2t) \right) \mathbf{k} = -\mathbf{k} = \boxed{\langle 0, 0, -1 \rangle}$$

$$\begin{aligned}\mathbf{r}'(t) \times \mathbf{r}''(t) &= \left\langle 0, 0, -8\cos^2(2t) - 8\sin^2(2t) \right\rangle = \langle 0, 0, -8 \rangle \\ \left\|\mathbf{r}'(t) \times \mathbf{r}''(t)\right\| &= 8\end{aligned}$$

$$\kappa = \frac{8}{2^3} = \boxed{1}, \quad a_T = \boxed{0}, \quad a_N = \frac{8}{2} = \boxed{4}$$

#22 (b)

$$\begin{aligned}\mathbf{r}(t) &= \left\langle \cos(t), \sin(t), t^2 \right\rangle \\ \mathbf{r}'(t) &= \left\langle -\sin(t), \cos(t), 2t \right\rangle \\ \mathbf{r}''(t) &= \left\langle -\cos(t), -\sin(t), 2 \right\rangle\end{aligned}$$

$$\mathbf{T}(t) = \frac{\left\langle -\sin(t), \cos(t), 2t \right\rangle}{\sqrt{1+4t^2}} = \boxed{\left\langle \frac{-\sin t}{\sqrt{1+4t^2}}, \frac{\cos t}{\sqrt{1+4t^2}}, \frac{2t}{\sqrt{1+4t^2}} \right\rangle}$$

$$\mathbf{N}(t) = \boxed{\frac{\left\langle -(1+4t^2)\cos t + 4t\sin t, -(1+4t^2)\sin t - 4t\cos t, 2\right\rangle}{\sqrt{16t^4+24t^2+5}}}$$

$$\mathbf{B}(t) = \boxed{\frac{\langle 2\cos t + 2t\sin t, 2\sin t - 2t\cos t, 1\rangle}{\sqrt{4t^2+5}}}$$

A comment about the preceding nasty computations: When doing beastly computations like the ones above it makes sense to check a bit as you go along. For example, I checked that $\mathbf{T}\cdot\mathbf{N} \equiv 0$ before I computed $\mathbf{B}$. To check my final answer, I checked that $\mathbf{T}\cdot\mathbf{B} \equiv 0$ & $\mathbf{N}\cdot\mathbf{B} \equiv 0$. Moving on...

$$\begin{aligned}
\mathbf{r}'(t) \times \mathbf{r}''(t) &= \langle 2\cos t + 2t\sin t, 2\sin t - 2t\cos t, 1\rangle \\
\left\|\mathbf{r}'(t) \times \mathbf{r}''(t)\right\| &= \sqrt{4t^2+5} \\
\mathbf{r}'(t)\cdot\mathbf{r}''(t) &= 4t
\end{aligned}$$

$$\kappa = \boxed{\frac{\sqrt{4t^2+5}}{(1+4+)^{3/2}}} \qquad a_T = \boxed{\frac{4t}{\sqrt{1+4t^2}}} \qquad a_N = \boxed{\sqrt{\frac{4t^2+5}{4t^2+1}}}$$