

Curves:

$$\vec{r}(t) = (x(t), y(t), z(t))$$

$$\text{Arclength } s(t) := \int_a^t \|\vec{r}'(u)\| \, du \quad (\text{measuring from } a \text{ to } t)$$

$$\text{Unit Tangent Vector } \vec{T}(t) := \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$$

$$\text{Curvature Vector } \frac{d\vec{T}}{ds} = \frac{\vec{T}'(t)}{\|\vec{r}'(t)\|}$$

$$\text{Curvature } \kappa := \left\| \frac{d\vec{T}}{ds} \right\|$$

$$\text{Principle Unit Normal } \vec{N}(t) := \frac{1}{\kappa} \frac{d\vec{T}}{ds} = \frac{\vec{T}'(t)}{\|\vec{T}'(t)\|}$$

$$\text{Binormal } \vec{B}(t) := \vec{T}(t) \times \vec{N}(t)$$

$$\text{Normal } \vec{N}(t) = \vec{B}(t) \times \vec{T}(t)$$

$$\vec{B}(t) = \frac{\vec{r}' \times \vec{r}''}{\|\vec{r}' \times \vec{r}''\|}$$

$$\vec{a}(t) = \frac{d^2s}{dt^2} \vec{T} + \left(\frac{ds}{dt} \right)^2 \kappa \vec{N} = a_T \vec{T} + a_N \vec{N}$$

$$a_T = \frac{\vec{r}' \bullet \vec{r}''}{\|\vec{r}'\|} \quad a_N = \frac{\|\vec{r}' \times \vec{r}''\|}{\|\vec{r}'\|^2} \quad \kappa = \frac{\|\vec{r}' \times \vec{r}''\|}{\|\vec{r}'\|^3}$$

For the curve given by the graph $y = f(x)$ we have:

$$\kappa(x) = \frac{|f''(x)|}{(1 + f'(x)^2)^{3/2}}.$$

For the plane curve given by $\vec{r}(t) = (x(t), y(t))$ we have:

$$\kappa(t) = \frac{|x'(t)y''(t) - x''(t)y'(t)|}{(x'(t)^2 + y'(t)^2)^{3/2}}.$$