Final Exam Cheat Sheet

I do not promise that I will give you all of this cheat sheet on the final... I promise that I will give you any part of it that I believe that you need. (e.g. to make everything fit onto two pages, I will surely not give you all of the curve formulas.)

**Integral Definitions and Basic Formulas:**

Line Integrals:

$$\int_C \vec{F}(\vec{r}) \bullet d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \bullet \vec{r}'(t)\, dt\,, \qquad \text{Orientation Matters!}$$

$$\int_C f(\vec{r})\, ds = \int_a^b f(\vec{r}(t))\, ||\vec{r}'(t)||\, dt\,, \qquad \text{Orientation Doesn't matter!}$$

$$\int_C f(\vec{r})\, dx = \int_a^b f(x(t), y(t), z(t))\, x'(t)\, dt\,, \qquad \text{Orientation Matters!}$$

Surface Integrals: With $\vec{N} := \vec{r}_u \times \vec{r}_v \neq 0$, and with $\vec{r}(R) = S$ we have

$$\int\int_S \vec{F} \bullet \vec{n}\, dS = \int\int_R \vec{F}(\vec{r}(u,v)) \bullet \vec{N}(u,v)\, du\, dv \qquad \text{Orientation Matters!}$$

$$\int\int_S f(\vec{r})\, dS = \int\int_R f(\vec{r}(u,v))\, ||\vec{N}(u,v)||\, du\, dv \quad \text{Orientation Doesn't matter!}$$

Green's Theorem: If $D$ is a region in the plane, and $\partial D$ has positive orientation (i.e. has counter-clockwise orientation), then

$$\int_{\partial D} P(x,y)\, dx + Q(x,y)\, dy = \int\int_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA\,.$$

Stokes's Theorem:

$$\int\int_S (\nabla \times \vec{F}) \bullet \vec{n}\, dA = \int_{\partial S} \vec{F} \bullet d\vec{r}\,.$$

(If $\vec{n}$ is pointing right at you then orient $\partial S$ in a positive fashion (i.e. counter-clockwise fashion, typically) to make the identity hold.)

**Spherical Coordinates:**

$$x = \rho \sin\phi \cos\theta \quad y = \rho \sin\phi \sin\theta \quad z = \rho \cos\phi$$

$$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta \ .$$

**Second Derivative Test:** Suppose the second partial derivatives of $f$ are continuous on a disk with center $(a,b)$ and suppose $\nabla f(a,b) = (0,0)$. Let

$$D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - f_{xy}(a,b)^2 \ .$$

(a) If $D > 0$ and $f_{xx}(a,b) > 0$, then $(a,b)$ is a local minimum.

(b) If $D > 0$ and $f_{xx}(a,b) < 0$, then $(a,b)$ is a local maximum.

(c) If $D < 0$, then $(a,b)$ is a saddle point.

**Curves:**

$$\vec{r}(t) = (x(t), y(t), z(t))$$

$$\begin{aligned}
\text{Arclength} \quad & s(t) := \int_a^t ||\vec{r}\,'(u)||\ du \quad \text{(measuring from } a \text{ to } t\text{)} \\
\text{Unit Tangent Vector} \quad & \vec{T}(t) := \frac{\vec{v}(t)}{||\vec{v}(t)||} = \frac{\vec{r}\,'(t)}{||\vec{r}\,'(t)||} \\
\text{Curvature Vector} \quad & \frac{d\vec{T}}{ds} = \frac{\vec{T}\,'(t)}{||\vec{v}(t)||} = \frac{\vec{T}\,'(t)}{||\vec{r}\,'(t)||} \\
\text{Curvature} \quad & \kappa := \left|\left|\frac{d\vec{T}}{ds}\right|\right| \\
\text{Principle Unit Normal} \quad & \vec{N}(t) := \frac{\vec{T}\,'(t)}{\left|\left|\vec{T}\,'(t)\right|\right|} \\
\text{Binormal} \quad & \vec{B}(t) := \vec{T}(t) \times \vec{N}(t)
\end{aligned}$$

$$\vec{B}(t) = \frac{\vec{r}\,' \times \vec{r}\,''}{||\vec{r}\,' \times \vec{r}\,''||}$$

$$\vec{a}(t) = \frac{d^2 s}{dt^2}\vec{T} + \left(\frac{ds}{dt}\right)^2 \kappa \vec{N} = a_T \vec{T} + a_N \vec{N}$$

$$a_T = \frac{\vec{r}\,' \bullet \vec{r}\,''}{||\vec{r}\,'||} \qquad a_N = \frac{||\vec{r}\,' \times \vec{r}\,''||}{||\vec{r}\,'||} \qquad \kappa = \frac{||\vec{r}\,' \times \vec{r}\,''||}{||\vec{r}\,'||^3}$$

Cheat Sheet Bonus:

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2},\ \sin^2\theta = \frac{1-\cos(2\theta)}{2},\ \sin(2\theta) = 2\sin\theta\cos\theta\ .$$