

Practice Midterm # 1 (The actual midterm will be shorter!)

Math 222, Fall 2020

You will not be allowed to use any type of calculator whatsoever, you will not be allowed to have any other notes, the test will be closed book, and there is no escape. The actual test will be graded **in red ink!** There will be no mercy for the weak. Mathematics is cumulative. Deal with it. What you don't know **will** hurt you. You need to be able to make simple and/or standard simplifications. In order to get credit or partial credit, your work must make sense.

I strongly suggest that you take this practice test under the conditions of the actual test! (Except that you might not do it all at once since it is longer than the test will be.)

1. Plot the points $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$ in a three-dimensional coordinate system. Label the axes. (Note: You should label everything in a way that makes the right hand rule correct!)
2. Which of the following are reasonable expressions? Which of them are not?

(a) $(\vec{u} \times \vec{v}) \bullet \vec{w}$

(b) $\vec{w} \bullet (\vec{u} \times \vec{v})$

(c) $\vec{u} \times (\vec{v} \bullet \vec{w})$

(d) $(\vec{v} \bullet \vec{w}) \times \vec{u}$

(e) $(\vec{u} \times \vec{v}) \bullet (\vec{w} \bullet \vec{x})$

(f) $\frac{\vec{u}}{\vec{w}}$

(g) $\frac{\vec{u}}{|\vec{w}|}$

(h) $3\vec{u} - 4\vec{w}$

(i) $|\vec{u} - 2\vec{w}| + |\vec{u} + 2\vec{w}|$

(j) $|3 - \vec{u}|$

The next few problems all refer to the following pairs of vectors:

(a) $\vec{u} := (3, 4)$ and $\vec{w} := (5, -1)$.

(b) $\vec{u} := (2, 5, -3)$ and $\vec{w} := (4, -1, 6)$.

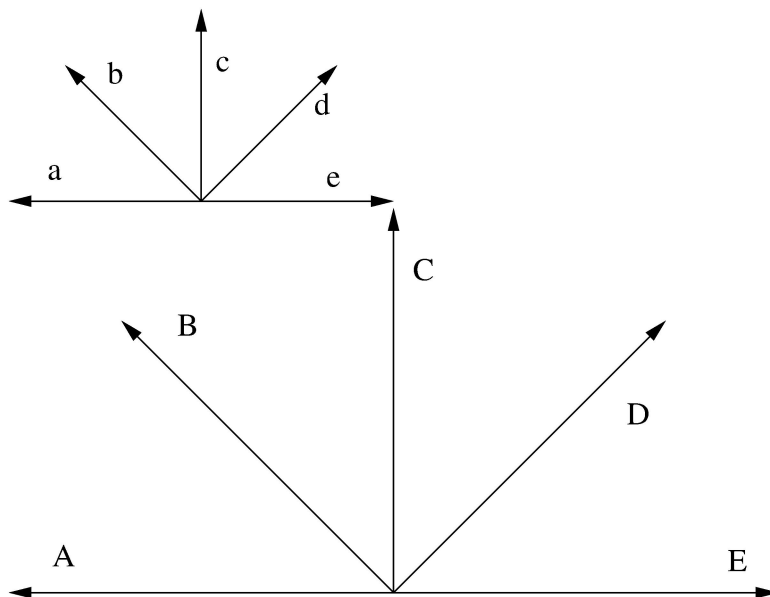
(c) $\vec{u} := 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{w} := 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$.

3. Find the norm of every vector above.
4. Find the distance from \vec{u} to \vec{w} for each pair of vectors above.
5. Find the dot products $\vec{u} \bullet \vec{w}$ and $\vec{w} \bullet \vec{u}$ and the cross products $\vec{u} \times \vec{w}$ and $\vec{w} \times \vec{u}$ for each pair of vectors above.
6. Find the area of the parallelogram spanned by \vec{u} and \vec{w} for each pair of vectors above.
7. Use dot products to write an exact formula for the angle between \vec{u} and \vec{w} for each pair above. (Note: It is NOT acceptable to need a calculator to know things like $\sin 0 = 0$ and $\cos 0 = 1$. You should be able to make a decent sketch of the sine and cosine functions.)
8. Use cross products to write an exact formula for the angle between \vec{u} and \vec{w} for each pair above. **Note: This question is slimy ... You can skip it ... On the other hand, you certainly need to know that**

$$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin \theta .$$

9. Find the volume of the parallelepiped spanned by:
 - (a) $\vec{a} := (1, 2, 3)$, $\vec{b} := (1, 3, 5)$, and $\vec{c} := (4, 1, -2)$.
 - (b) $\vec{a} := \hat{\mathbf{i}} - \hat{\mathbf{k}}$, $\vec{b} := \hat{\mathbf{j}} + \hat{\mathbf{k}}$, and $\vec{c} := \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$.
10. Give the parametric equation and the symmetric equations for the line passing through the points $(2, 3, 4)$ and $(-2, -5, 1)$.
11. Give an expression for the plane containing the line from the previous problem and the point $(1, -1, 2)$.
12. Give an expression for the plane which contains the point $(2, 3, 4)$ and which is perpendicular to the vector $(-1, -4, 7)$.
13. Find an expression for the plane which contains the points $(5, 6, 7)$, $(1, 2, -1)$, and $(3, -1, 2)$.

14. Give an expression for the sphere with center $(2, -4, -7)$ and radius equal to 5.



For the next few problems we refer to the figure above which is drawn to scale. The vectors a , b , c , d , and e all have unit length, and the vectors A , B , C , D , and E all have length two. All of the angles between the vectors are multiples of 45 degrees.

15. Put the numbers in order from smallest to largest:
- $|A|$, $|a|$, $|A - E|$, $|C - E|$, and $|d - e|$.
 - $a \bullet e$, $a \bullet d$, $a \bullet c$, $a \bullet b$, and $a \bullet a$.
 - $|a \times e|$, $|d \times a|$, $|a \times c|$, and $|A \times C|$.
16. Assuming that you are viewing this page normally, which of the following vectors is pointing toward you?
- $A \times B$.
 - $D \times C$.

(c) $A \times D$.

(d) $E \times B$.

17. Compute the following explicitly:

(a)

$$E \bullet A .$$

(b)

$$B \bullet B .$$

(c)

$$|a \times c| .$$

(d)

$$|A \times C| .$$

(e)

$$|a \times a| .$$

(f)

$$C \bullet A .$$

(g)

$$\frac{D \bullet E}{d \bullet e} .$$

(h)

$$\frac{C \bullet B}{C \bullet b} .$$

18. Find the shortest distance from the point $(3, 1, -2)$ to the plane given by $2x - 3y + z = 5$.

19. Familiarize yourself again with quadric surfaces, redo the matchup problem that you did with HW3, and maybe look at some graphs on Wolfram Alpha. (If you are unfamiliar with Wolfram Alpha, then google it.) You should be able to do a sketch of an ellipsoid, a paraboloid, and of course, a sphere when given only the equation for it.

20. You should be able to write down a parametrization for a circle or an ellipse.

21. Make the following definitions

(a) $f(t) := e^{2t}$, $g(t) := \sin(3t)$.

(b) $\vec{u}(t) := (5t, e^{-3t}, \cos(7t))$, $\vec{w}(t) := (e^t, \sin(4t), t^8)$.

Now with these definitions, compute the following:

(a)

$$\frac{d}{dt} (f(t)\vec{u}(t) + g(t)\vec{w}(t)) .$$

(b)

$$\frac{d}{dt} (\vec{u}(t) \bullet \vec{w}(t)) .$$

(c)

$$\frac{d}{dt} (\vec{u}(t) \times \vec{w}(t)) .$$

(d)

$$\frac{d}{dt} (\vec{u}(f(t))) .$$

22. Find the unit tangent, unit normal, unit binormal, the curvature, and the tangential and normal components of acceleration for the following curves:

(a) $\vec{r}(t) := (\sin(2t), \cos(2t), 7)$.

(b) $\vec{r}(t) := (\cos(t), \sin(t), t^2)$.