Practice Midterm # 2 (The actual midterm will be shorter!)

Math 222, Fall 2018

You will not be allowed to use any type of calculator whatsoever, you will not be allowed to have any other notes, the test will be closed book, and there is no escape. The actual test will be graded **in red ink!** There will be no mercy for the weak. Mathematics is cummulative. Deal with it. What you don't know **will** hurt you. You need to be able to make simple and/or standard simplifications. In order to get credit or partial credit, your work must make sense.

I strongly suggest that you take this practice test under the conditions of the actual test! (Except that you might not do it all at once since it is longer than the test will be.)

1. **Ignore this question!** Find the unit tangent, unit normal, unit binormal, the curvature, and the tangential and normal components of acceleration for the following curves:

   (a) $\vec{r}(t) := (\sin(2t), \cos(2t), 7)$.

   (b) $\vec{r}(t) := (\cos(t), \sin(t), t^2)$.

2. **Ignore this question!** Section 14.1 Problems 30, 55-60

3. State the definition of continuity at a point.

4. Compute all of the first derivatives for the following functions:

   (a) $f(x, y) = x^2 \sin(3xy^2 + 4x^2y)$.

   (b) $g(x, y) = x^3y^2e^{x^5y^4}$.

   (c)
   $$h(x, y, z) = \frac{(xy^2 - z^3)\sin(x^4 - y^5z)}{\ln(2 + x^2z^2 + y^2)} .$$

   (d) $F(w, x, y, z) = w^7x^5y^3z - w^2x^4 + y^6z^8 + w + 2x + 3y + 4z + \pi$.

5. Compute all of the second derivatives for the following functions:

   (a) $f(x, y) = x^2 \sin(xy^2)$.

   (b) $g(x, y) = x^3y^2 - 4x^2y^3 + 3x^7y^9$.

   (c) $h(x, y, z) = \sin(xy^2z^3)$.

(d) $F(x, y, z) = e^{(xy+2xz+3yz)}$.

6. Find the tangent plane and linear approximation to the given function at the given point:

   (a) $f(x, y) = x^2y^3$ at $(3, -1)$.

   (b) $g(x, y) = 3xy^2 - 4x^2y$ at $(2, 3)$.

7. Find the tangent plane at the given point:

   (a) $x^2 + y^4 + z^6 = 26$ at $(3, -2, 1)$.

   (b) $xy + 2xz + 3yz = 14$ at $(-2, 3, 4)$.

8. Write down the chain rule for the following situations:

   (a) What is $\frac{\partial f}{\partial s}$ if f is a function of x, y, and z, and x, y, and z are each functions of s and t?

   (b) What is $\frac{df}{dt}$ if f is a function of w, x, y, and z, and w, x, y, and z are each functions of t?

9. For the function $f(x, y) = x + 3x^2y$, what is the gradient at each of the following points?

   (a) $(-2, 3)$.

   (b) $(-1, 4)$.

   (c) $(5, 0)$.

10. For the function and points from the previous question answer all of the following:

    (a) Which is the direction that the function is increasing the fastest?

    (b) What is the rate of change in the direction $(3, 4)$?

11. Find the distance from the point $(1, 2, 3)$ to the plane

$$2(x - 1) - 3(y + 2) + 4(z - 5) = 0$$

in three different ways:

   (a) Without Calculus. (i.e. The same way you would have done it on the first test.)

(b) By making a reasonable substitution and then minimizing an appropriate function of two variables.

(c) By using Lagrange Multipliers.

12. Find and classify all of the critical points of $f(x, y) = 16x^3 + 2xy^2 + 20x^2 + y^2$.

13. **Ignore this question!** Find the absolute maximum and absolute minimum of the function $F(x, y) = 3x - x^3 - 2y^2 + y^4$ on the set $-2 \leq x \leq 2,\ -2 \leq y \leq 2$.

14. Set up, **but do not solve** the following Lagrange Multipliers problems. Be sure to list the unknowns and the equations you need to use to find them.

    (a) Minimize: $F(x, y, z) = xe^{2y+3z}$.
    Subject to the constraint: $G(x, y, z) = x^4 + y^4 + z^4 - xy + 2xz - 3yz = 40$.

    (b) Maximize: $F(x, y, z) = x^2y^3 + x^3y^2 - z$.
    Subject to the constraint: $G(x, y, z) = x^2 + (2y+1)^2 + (3z-1)^2 = 7^2$.

    (c) Minimize: $F(x, y, z) = xye^{yz}$.
    Subject to the constraints: $G(x, y, z) = x^2 + y^2 + z^2 = 5^2$, and $H(x, y, z) = xy + 2xz - yz = 10$.

15. Find the maximum and the minimum of the function $x^2 - 2x + y^2 - 4y$ in the region $(x - 4)^2 + (y + 3)^2 \leq 8^2$.

16. From HW #7, review questions 1 - 5 in particular.

17. From HW #8, review all four questions.