

Practice Midterm # 3 (The actual midterm will be shorter!)

Math 222, Fall 2010

You will not be allowed to use any type of calculator whatsoever, you will not be allowed to have any other notes, the test will be closed book, and there is no escape. The actual test will be graded **in red ink!** There will be no mercy for the weak. Mathematics is cumulative. Deal with it. What you don't know **will** hurt you. You need to be able to make simple and/or standard simplifications. In order to get credit or partial credit, your work must make sense.

I strongly suggest that you take this practice test under the conditions of the actual test! (Except that you might not do it all at once since it is longer than the test will be.)

1. Compute the following:

(a)

$$\int_0^3 \int_0^4 (xy^2 - x^2y) dx dy .$$

(b)

$$\int_0^2 \int_0^5 \int_{-1}^0 xyz dz dy dx .$$

2. Compute the following:

(a) Let T be the triangle with vertices $(0, 0)$, $(1, 0)$, and $(2, 1)$.

$$\iint_T 12xy dA .$$

(b) Let T be the tetrahedron with vertices $(0, 0, 0)$, $(2, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 6)$.

$$\iiint_T x dV .$$

(c) Let R be the region in the first quadrant between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

$$\iint_R x dA .$$

(d) Let R be the region in the first octant bounded by $x^2 + y^2 + z^2 = 16$.

$$\int \int \int_R z \, dV .$$

3. Set up the following as iterated integrals. Although you do not need to compute these integrals you should set them up in an intelligent way. For example, if the domain is a ball centered at the origin, and the integrand is a function of the distance to the origin, then you should use spherical coordinates.

(a) Let T be the tetrahedron with vertices $(0, 0, 0)$, $(2, 0, 0)$, $(2, 4, 0)$, and $(2, 4, 1)$.

$$\int \int \int_T x \, dV .$$

(b) Let R be the region in the first octant determined by the inequalities

$$36 \leq x^2 + y^2 + z^2 \leq 144 \quad \text{and} \quad z \leq \sqrt{x^2 + y^2} .$$

$$\int \int \int_R x \, dV .$$

(c) Let R be the region determined by the inequalities:

$$x \geq 0, \quad x^2 + y^2 \leq 16, \quad -x \leq z \leq x^2 + y^2 .$$

$$\int \int \int_R z \, dV .$$

(d) Let R be the region determined by the inequalities:

$$x \geq 0, \quad y \geq 0, \quad z \geq 0, \quad x^2 + z^2 \leq 4, \quad x + y \leq 8 .$$

$$\int \int \int_R (x + y + z) \, dV .$$

4. Use Fubini's Theorem to evaluate the following integrals. (In other words, start by changing the order of integration.)

(a)

$$\int_{y=0}^2 \int_{x=\frac{y}{2}}^1 e^{x^2} \, dx \, dy .$$

(b)

$$\int_{y=0}^6 \int_{x=\frac{y^2}{9}}^4 e^{x^{3/2}} dx dy .$$

5. Write the integral as five other iterated integrals in Cartesian coordinates:

(a)

$$\int_{z=0}^2 \int_{y=0}^{\frac{6-3z}{2}} \int_{x=0}^{6-3z-2y} \rho(x, y, z) dx dy dz .$$

(b)

$$\int_{z=0}^2 \int_{y=-2}^6 \int_{x=0}^{(z-2)^2} \rho(x, y, z) dx dy dz .$$

Important hint for this problem: I accidentally did something a little bit evil... You will want to replace $(z - 2)^2$ with $(2 - z)^2$ before taking the square root when changing to integrate z before x .

6. Find the curl and the divergence of the following vector fields:

(a)

$$\vec{F} = (x + 2y - 3z, y + 2z - 3x, z + 2x - 3y) .$$

(b)

$$\vec{G} = (y^2 - z^2, z^2 - x^2, x^2 - y^2) .$$

(c)

$$\vec{H} = (x \sin(y), y \cos(x), z^2) .$$

7. For the following vector fields, determine if they are conservative, and if so, find a corresponding potential function.

- One of the $\vec{F}, \vec{G}, \vec{H}$ below was supposed to be:

$$\vec{\Theta} = (2x \cos(xy) - x^2 y \sin(xy), -x^3 \sin(xy)) .$$

Oops... Do that one instead and skip at least one of $\vec{F}, \vec{G}, \vec{H}$ to save time.

(a)
$$\vec{F} = (2x \cos(xy) + x^2y \sin(xy), -x^3 \cos(xy)) .$$

(b)
$$\vec{G} = (2x \cos(xy) - x^2y \sin(xy), x^3 \cos(xy)) .$$

(c)
$$\vec{H} = (2x \cos(xy) - x^2y \sin(xy), -x^3 \cos(xy)) .$$

(d)
$$\vec{K} = (yz + 1, xz + 2y, xy + 3z^2) .$$

- One of $\vec{F}, \vec{G}, \vec{H}$ was supposed to be:

$$\vec{\Theta} = (2x \cos(xy) - x^2y \sin(xy), -x^3 \sin(xy)) .$$

8. What does it mean if a vector field is incompressible? What does it mean if it is irrotational? What does it mean if it is conservative? What is a potential function? Are any of these concepts linked?