Practice Final

Math 222, Fall 2018

Here is practice for the new material only! For the old material, you need to study the old practice exams!

You will not be allowed to use any type of calculator whatsoever, you will not be allowed to have any other notes, the test will be closed book, and there is no escape. The actual test will be graded **in red ink!** There will be no mercy for the weak. Mathematics is cummulative. Deal with it. What you don't know **will** hurt you. You need to be able to make simple and/or standard simplifications. (Especially where I emphasize!) In order to get credit or partial credit, your work must make sense.

I strongly suggest that you take this practice test under the conditions of the actual test! (Except that you might not do it all at once since it is longer than the test will be.)

1. State the divergence theorem.

2. State the fundamental theorem of line integrals.

3. Suppose that a tetrahedral T has vertices:

$$(0,-1,0),\ (1,0,0),\ (0,1,0),\ (0,0,1)\ .$$

Assuming that you want to express the integral:

$$\int\int\int_T e^{(x+y+z)}\ dV$$

as a single iterated integral, then which orders of integration will work best?

4. Express the following integrals as iterated integrals on a subset of $\mathbb{R}^1$, $\mathbb{R}^2$, or $\mathbb{R}^3$. If it would make the computation simpler and if it applies, then use Green's Theorem, The Divergence Theorem, or The Fundamental Theorem of Line Integrals. You cannot necessarily compute the resulting integrals, so do not try unless the answer is obvious.

For all of the following problems, let:

$$\vec{F}(x,y,z) = (x, 2x+yz, y^2-z^2) \qquad \vec{G}(x,y,z) = (yz, xz+3y^2, xy+2z)$$

$$f(x,y,z) = 1 + x^2e^x + y^4e^{2y} + z^6e^{3z} \qquad g(x,y,z) = xyz\ .$$

(a) Let C be the curve $\{\ x^2 + y^2 = 25,\ z = 0\}$. Assume that C has counter-clockwise orientation when viewed from above. Set up:

$$\int_C \vec{F} \bullet d\vec{r}\,, \quad \text{and} \quad \int_C \vec{G} \bullet d\vec{r}\,.$$

(b) Now let C be the curve $\{\ x = 4,\ \frac{y^2}{9} + \frac{z^2}{16} = 1\}$. Assume that C has counter-clockwise orientation when viewed from the origin. Set up the same line integrals as before but with the new definition of C.

(c) Let C be the part of the parabola $y = x^2 + 1$, with $z \equiv 3$ with starting point $(-2, 5, 3)$ and ending point $(4, 17, 3)$. Same line integrals.

(d) Let C be the line segment from $(1, 0, 1)$ to $(0, 1, 0)$. Set up:

$$\int_C f(x,y,z)\ dx + g(x,y,z)\ dy \quad \text{and} \quad \int_C f(x,y,z)\ ds\,.$$

(e) Let C be the piece of the hyperbola $xy = 4, z = 5$ where the x-value starts at 1 and finishes at 4. Set up the same line integrals as the last problem.

(f) Let E be the surface below the xy-plane (i.e. $z \leq 0$) created by intersecting the sphere $x^2+y^2+z^2 = 4$ with the solid cylinder $x^2 + y^2 \leq 1$. Let $\vec{n}$ be the unit normal to E with positive $\hat{k}$ component. Set up:

$$\int\int_E \vec{F} \bullet \vec{n}\ dS \quad \text{and} \quad \int\int_E \vec{G} \bullet \vec{n}\ dS\,.$$

(g) Now let E be the part of the sphere $x^2 + y^2 + z^2 = 16$ which lies above the plane $z = -2$, and let $\vec{n}$ be the unit normal which points out of the sphere. Set up the same surface integrals as in the last problem.

(h) Let E be the boundary of the tetrahedral, T, given by the inequalities:

$$x \geq 0,\ y \geq 0,\ z \geq 0,\ 2x + 3y + 5z \leq 30\,.$$

Let $\vec{n}$ be the exterior unit normal. Set up the same surface integrals as in the last problem.

(i) Let E be the boundary of the bounded region trapped between the xy-plane and the graph of $z = 25 - x^2 - y^2$. Let $\vec{n}$ be the exterior unit normal again. Same surface integrals.

(j) Let E be given by the graph $z = e^x(\sin y + 5)$ for (x, y) satisfying $-1 \leq x \leq 5$ and $-4 \leq y \leq 2$. Let $\vec{n}$ be the unit normal with positive z-component. Same integrals.

(k) Let E be the intersection of $z = 2\sqrt{x^2 + y^2}$ with $z \leq 18$. Set up

$$\int\int_E f(x, y, z)\; dS \quad \text{and} \quad \int\int_E g(x, y, z)\; dS\;.$$

(l) Let E be the torus obtained by taking the circle in the xz-plane given by $(x - 5)^2 + z^2 = 4$ and rotating it around the z-axis. Set up the same integrals as in the last problem.

(m) Let E be the set given by $\{0 \leq x \leq 4,\; y^2 + z^2 = 25\}$. Same integrals.

(n) Let E be the part of the ellipsoid given by

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 1$$

which is above the xy-plane. Same integrals.