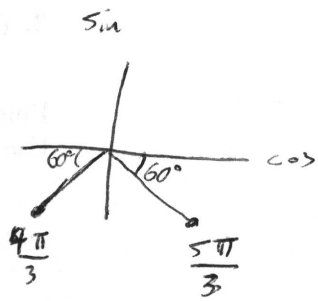
Recit #10

Tues, Mar 29, 2022

$\widetilde{1-3}$

$$\sin\Big(\underbrace{5x+\frac{\pi}{4}}_{\text{set as } \theta}\Big) = -\frac{\sqrt{3}}{2}$$

consider $\sin\theta = -\frac{\sqrt{3}}{2}$



θ is 60° in quadrants where $\sin(-)$ is negative

so solutions are $\theta = \frac{4\pi}{3} + 2\pi k,\ \frac{5\pi}{3} + 2\pi k.$

Now bring back the subexpression involving x, and solve.

$$\begin{cases} 5x + \frac{\pi}{4} = \frac{4\pi}{3} + 2\pi k \\ 5x + \frac{\pi}{4} = \frac{5\pi}{3} + 2\pi k \end{cases}$$

$$\frac{4\pi}{3} - \frac{\pi}{4} = \frac{16\pi}{12} - \frac{3\pi}{12} = \frac{13\pi}{12}$$

$$\frac{5\pi}{3} - \frac{\pi}{4} = \frac{20\pi}{12} - \frac{3\pi}{12} = \frac{17\pi}{12}$$

$$\Rightarrow \begin{cases} 5x = \frac{13\pi}{12} + 2\pi k \\ 5x = \frac{17\pi}{12} + 2\pi k \end{cases}$$

$$\Rightarrow \boxed{x = \frac{13\pi}{60} + \frac{2\pi}{5}k,\ \frac{17\pi}{60} + \frac{2\pi}{5}k,\quad k \in \mathbb{Z}}$$

5) $4\sin^2 y = \cos y - 1$

Rewrite $\sin^2 y$ in terms of $\cos y$ using Pythag identity

$$\sin^2 + \cos^2 = 1$$

$$\Rightarrow \sin^2 = 1 - \cos^2$$

$$\Rightarrow 4(1-\cos^2 y) = \cos y - 1$$

$$\Rightarrow 4 - 4\cos^2 y - \cos y + 1 = 0$$

$$\Rightarrow 4\cos^2 y + \cos y - 5 = 0$$

this is $4x^2 + x - 5 = 0$, letting $x = \cos y$

This factors

$$(4x+5)(x-1) = 0$$

Otherwise use quadratic formula to get factorization.

$$\Rightarrow (4\cos y + 5)(\cos y - 1) = 0$$

$\Rightarrow 4\cos y + 5 = 0$ OR $\cos y - 1 = 0$

$\cos y = \frac{-5}{4}$ | $\cos y = 1$

Impossible | $y = 2\pi k,\ k \in \mathbb{Z}$

$$\Rightarrow \boxed{y = 2\pi k,\ k \in \mathbb{Z}}$$

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$\sin u \cos u + \sin u - \cos u - 1 = 0$ ← Crucial step is to spot this factorizes.

$\Rightarrow (\sin u - 1)(\cos u + 1) = 0$

$\Rightarrow \sin u - 1 = 0$ OR $\cos u + 1 = 0$

$\sin u = 1$ | $\cos u = -1$

$u = \frac{\pi}{2} + 2\pi k$ | $u = \pi + 2\pi k$

$\Rightarrow \boxed{u = \frac{\pi}{2} + 2\pi k, \quad \pi + 2\pi k, \quad k \in \mathbb{Z}}$

$$\vec{v}_1 \cdot \vec{v}_2 = x_1 x_2 + y_1 y_2$$
$$\vec{v}_1 \wedge \vec{v}_2 = x_1 y_2 - x_2 y_1$$

8–9

$$\vec{v}_1 = \begin{bmatrix} -5 \\ -7 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$\vec{v}_1 \cdot \vec{v}_2 = (-5)(3) + (-7)(-2)$$
$$= -15 + 14 = \boxed{-1}$$

$$\vec{v}_1 \wedge \vec{v}_2 = (-5)(-2) - (3)(-7)$$
$$= 10 + 21 = \boxed{31}$$

10–11

$\|\vec{v}_1\| = 3, \ \|\vec{v}_2\| = 6, \ \|\vec{v}_1 - \vec{v}_2\| = 5.$ Find $\vec{v}_1 \cdot \vec{v}_2$

$$\|\vec{v}_1 - \vec{v}_2\|^2 = \|\vec{v}_1\|^2 + \|\vec{v}_2\|^2 - 2(\vec{v}_1 \cdot \vec{v}_2)$$ ← generalized Pythagorean theorem

$$5^2 = 3^2 + 6^2 - 2(\vec{v}_1 \cdot \vec{v}_2)$$ Plug in values

$$\Rightarrow 25 - 9 - 36 = -2(\vec{v}_1 \cdot \vec{v}_2)$$
$$-20 = -2(\vec{v}_1 \cdot \vec{v}_2)$$
$$\vec{v}_1 \cdot \vec{v}_2 = \boxed{10}$$

$\boxed{12-13}$ $\quad v_1 = \begin{bmatrix} -\sqrt{3} \\ 5 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ -2\sqrt{3} \end{bmatrix}$

Find geometric angle & both turning angles.

Exact values.

Geometric angle

$$\boxed{\gamma} = \arccos\left(\frac{v_1 \cdot v_2}{\|v_1\| \, \|v_2\|}\right)$$

$$= \arccos\left(\frac{-14\sqrt{3}}{4 \cdot 7}\right)$$

$$= \arccos\left(\frac{-\sqrt{3}}{2}\right)$$

$$= \boxed{\frac{5\pi}{6}}$$

calculator or unit circle

output of arccos

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

30°

so 30° in quad II

30°

aka $\frac{5\pi}{6}$

$$v_1 \cdot v_2 = (-\sqrt{3})(4) + (5)(-2\sqrt{3})$$

$$= -4\sqrt{3} - 10\sqrt{3}$$

$$= -14\sqrt{3}$$

$$\|v_1\| = \sqrt{(-\sqrt{3})^2 + (5)^2}$$

$$= \sqrt{3 + 25} = \sqrt{28} = 2\sqrt{7}$$

$$\|v_2\| = \sqrt{(4)^2 + (-2\sqrt{3})^2}$$

$$= \sqrt{16 + 12} \quad 2\sqrt{28} = 2\sqrt{7}$$

$$v_1 \wedge v_2 = (-\sqrt{3})(-2\sqrt{3}) - 5(4)$$

$$= 6 - 20 = -14$$

Turning angles

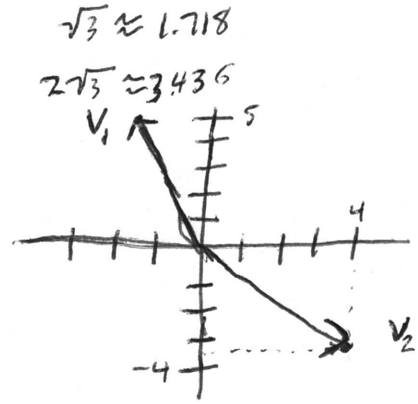
$$\boxed{\tau_{v_1 \text{ over } v_2} = \operatorname{sign}(v_1 \wedge v_2) \cdot \gamma = -\frac{5\pi}{6}}$$

$$\boxed{\tau_{v_2 \text{ over } v_1} = \frac{5\pi}{6}}$$

or can see these two visually:

$\sqrt{3} \approx 1.718$

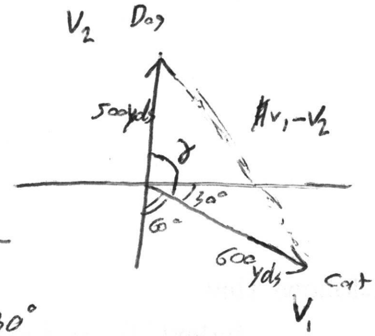
$2\sqrt{3} \approx 3.436$



$\boxed{\tilde{14}}$ • cat runs 600 yds in S60°E

• dog runs 500 yds North.

Find distance b/t pets. Round 2 decimal places.



Let v_1 = cat, v_2 = dog.

$\gamma = 90° + 30°$
$= \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

Want $\|v_1 - v_2\|$.

Generalized pythag:

$$\|v_1 - v_2\|^2 = \|v_1\|^2 + \|v_2\|^2 - 2(\underbrace{v_1 \cdot v_2})$$

$$= \|v_1\| \cdot \|v_2\| \cos \gamma$$

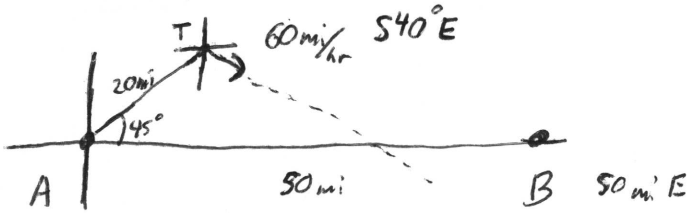
$$= 600 \cdot 500 \cos \frac{2\pi}{3}$$

$$= 300{,}000 \cdot \left(-\frac{1}{2}\right)$$

$$\|v_1 - v_2\| = \sqrt{600^2 + 500^2 + 300{,}000}$$

$$\approx \boxed{953.94 \text{ yds}}$$

$\boxed{15}$

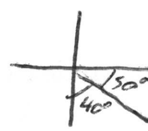


Find distance(T, B) after $\frac{1}{2}$ hr.

Use coordinates.
Let $A = (0,0)$, so $B = (50,0)$

- $T_{init} = 20 \cdot \vec{u}_{45°} = 20\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 10\sqrt{2} \\ 10\sqrt{2} \end{bmatrix}$ ← initial position.

- Direction of travel: S 40° E



I prefer this notation for vectors

Unit direction vector: $\vec{u}_{-50°} = \langle \cos(-50°), \sin(-50°) \rangle$

- Total travel in $\frac{1}{2}$ hr: $30\,\text{mi} \cdot \vec{u}_{-50°} = \langle 30\cos(-50°), 30\sin(-50°) \rangle$
$\simeq \langle 19.2836, -22.9813 \rangle$ mi

- $T_{\frac{1}{2}hr} = T_{init} + \text{travel}$
$\approx \langle 33.4258, -8.8392 \rangle$ ← position of tornado after $\frac{1}{2}$ hr.

- $\text{dist}(T_{\frac{1}{2}hr}, B) = \sqrt{(50 - 33.4258)^2 + (0 - (-8.8392))^2}$ ← distance formula.
$\approx \boxed{18.78 \text{ mi}}$