

Revit 12
Apr 12, 2022

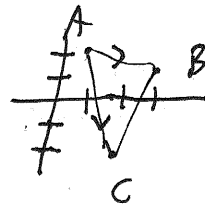
1-2 Area of parallelogram formed by

$$\vec{v}_1 = \begin{bmatrix} -3 \\ 5 \end{bmatrix} \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{aligned} \text{Area}(\mathcal{P}) &= |v_1 \wedge v_2| \\ &= |-3(2) - (5)(1)| \\ &= |-6 - 5| = \boxed{11} \end{aligned}$$

3-4 Find area of triangle $\triangle ABC$
(Assume units are ft.)

$$A = (1, 2) \quad B = (3, 1) \quad C = (-2, -2)$$



$$\vec{AB} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\vec{AC} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$$\text{Area}(\triangle ABC) = \frac{1}{2} | \vec{AB} \wedge \vec{AC} |$$

$$= \frac{1}{2} | 2(-4) - (-1)(1) |$$

$$= \frac{1}{2} | -8 + 1 | = \boxed{\frac{7}{2} \text{ ft}^2}$$

5-6) Find orthogonal projection $\vec{p} = \text{proj}_{\vec{w}}(\vec{v})$

for
 $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \vec{w} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$$\vec{v} \cdot \vec{w} = 2(3) + (3)(1) = 9$$

$$\vec{w} \cdot \vec{w} = 3 \cdot 3 + 1 \cdot 1 = 10$$

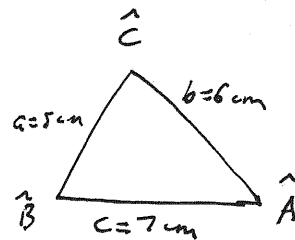
$$\text{proj}_{\vec{w}}(\vec{v}) = \left(\frac{\vec{v} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} \right) \vec{w}$$

$$= \frac{9}{10} \vec{w}$$

$$= \begin{bmatrix} \frac{27}{10} \\ \frac{9}{10} \end{bmatrix}$$

7-14) Solve triangle. Round to 0.001

a) $a = 5 \text{ cm}, b = 6 \text{ cm}, c = 7 \text{ cm}$



SSS case

$$\hat{A} = \arccos\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$= \arccos\left(\frac{5}{7}\right) = \frac{0.5441099}{0.775193} \text{ or } \frac{0.477127}{0.4153} \quad 0.775193$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \sin B = \frac{b}{a} \sin A = \frac{6}{5} \sin(0.775193)$$

$$\sin B = 0.839825$$

$$B = \frac{0.66516}{0.996961} \text{ or } \frac{0.7711}{0.571217} \quad 0.996961 \text{ or } 0.571217$$

~~$$\frac{\sin A}{a} = \frac{\sin C}{c}$$~~

$$\hat{A} + \hat{B} + \hat{C} = 180^\circ$$

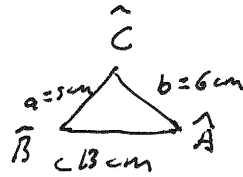
$$\Rightarrow \hat{C} \approx \pi - 0.775193 - 0.996961$$

$$\approx 1.36944 \text{ or } 78.4608^\circ$$

$$78.4631^\circ \text{) round off error}$$

b) SSS case Notice $c > a+b$.

$a = 5 \text{ cm}$ $b = 6 \text{ cm}$ $c = 13 \text{ cm}$



$$\hat{A} = \arccos\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

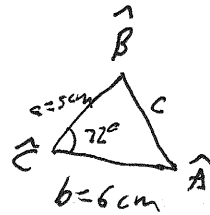
$$= \arccos\left(\frac{15}{13}\right)$$

$\curvearrowright > \dagger$

No soln

c) ASAS

$a = 5 \text{ cm}$ $b = 6 \text{ cm}$ $\hat{C} = 72^\circ$



$$c^2 = a^2 + b^2 - 2ab \cos \hat{C}$$

$$\Rightarrow \boxed{c = 6.51606}$$

$$\frac{\sin \hat{C}}{c} = \frac{\sin \hat{A}}{a} \Rightarrow \sin \hat{A} = \frac{a}{c} \sin \hat{C}$$

$$\hat{A} = \arcsin\left(\frac{a}{c} \sin \hat{C}\right)$$

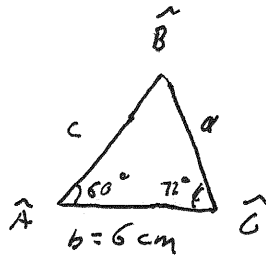
$$\boxed{\hat{A} = 0.817998 \text{ or } 46.8678^\circ}$$

$$\hat{B} = 180^\circ - \hat{A} - \hat{C}$$

$$\boxed{\hat{B} = 61.1322^\circ}$$

d) ASA case

$$\hat{A} = 60^\circ, b = 6 \text{ cm}, \hat{C} = 72^\circ$$



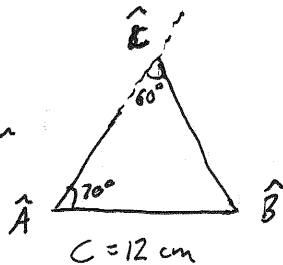
$$\hat{B} = 180^\circ - 72^\circ - 60^\circ = \boxed{48^\circ}$$

$$\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b} \Rightarrow \boxed{a} = \frac{b \sin \hat{A}}{\sin \hat{B}} = \boxed{6.99211 \text{ cm}}$$

$$\frac{\sin \hat{C}}{c} = \frac{\sin \hat{B}}{b} \Rightarrow \boxed{c} = \frac{b \sin \hat{C}}{\sin \hat{B}} = \boxed{7.67864 \text{ cm}}$$

e) AAS case $\hat{A} = 70^\circ$ $\hat{C} = 60^\circ$ $c = 12 \text{ cm}$

$$\hat{B} = 180^\circ - 70^\circ - 60^\circ = \boxed{50^\circ}$$

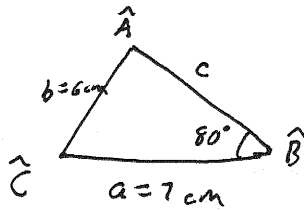
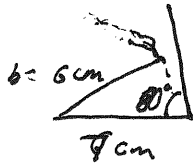


$$\frac{\sin \hat{C}}{c} = \frac{\sin \hat{B}}{b} \Rightarrow b = \frac{c \sin \hat{B}}{\sin \hat{C}} = \boxed{10.6146 \text{ cm}}$$

$$\frac{\sin \hat{C}}{c} = \frac{\sin \hat{A}}{a} \Rightarrow a = \frac{c \sin \hat{A}}{\sin \hat{C}} \approx \boxed{13.0208 \text{ cm}}$$

f) SSA No solution case

$a = 7 \text{ cm}$ $b = 6 \text{ cm}$ $\hat{B} = 80^\circ$



$$\frac{\sin \hat{B}}{b} = \frac{\sin \hat{A}}{a}$$

$$\Rightarrow \sin A = \frac{a}{b} \sin \hat{B}$$

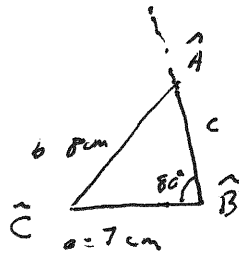
$$\Rightarrow \hat{A} = \arcsin\left(\frac{a}{b} \sin \hat{B}\right) = \arcsin(1.14894)$$

impossible

No soln

g) SSA 1 soln case

$a = 7 \text{ cm}$ $b = 8 \text{ cm}$ $\hat{B} = 80^\circ$



$$\frac{\sin \hat{B}}{b} = \frac{\sin \hat{A}}{a} \Rightarrow \sin \hat{A} = \frac{a}{b} \sin \hat{B}$$

$$\Rightarrow \hat{A} = \arcsin\left(\frac{a}{b} \sin \hat{B}\right) = 1.03862 \text{ or } 59.5074^\circ$$

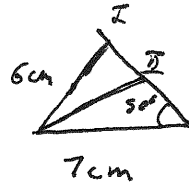
$$\hat{C} = 180^\circ - 80^\circ - 59.5074^\circ = 40.4926^\circ$$

$$\frac{\sin \hat{B}}{b} = \frac{\sin \hat{C}}{c} \Rightarrow c = \frac{b \sin \hat{C}}{\sin \hat{B}} = 5.27494 \text{ cm}$$

alt: $180^\circ - 59.5074^\circ = 120.4926^\circ$
 but $a < b \Rightarrow A < B$
 so reject alt angle.

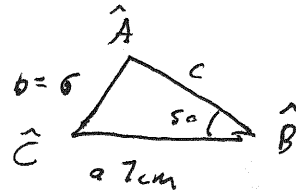
h) SSA ~~case~~ 2 solns case

$$a = 7 \text{ cm} \quad b = 6 \text{ cm}, \quad \hat{B} = 50^\circ$$



$$\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b}$$

$$\sin \hat{A} = \frac{a}{b} \sin \hat{B}$$



$$\hat{A} = \arcsin(0.823719)$$

$$= 1.10557 \Rightarrow \boxed{63.3462^\circ}$$

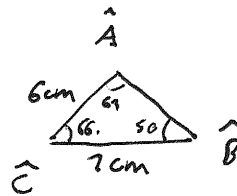
$$\text{alt: } 180^\circ - 63.3462^\circ = \boxed{116.654^\circ}$$

Both angles possible

$$a > b \Rightarrow \hat{A} > \hat{B}$$

(I) Case $\hat{A} = \boxed{63.3462^\circ}$

$$\hat{C} = 180^\circ - 50^\circ - 63.3462^\circ = \boxed{66.6538^\circ}$$



~~side~~

$$\frac{\sin \hat{C}}{c} = \frac{\sin \hat{B}}{b} \Rightarrow \boxed{c} = b \frac{\sin \hat{C}}{\sin \hat{B}} = \boxed{7.19118 \text{ cm}}$$

(II) Case $\hat{A} = \boxed{116.654^\circ}$

$$\hat{C} = 180^\circ - 50^\circ - 116.654^\circ = \boxed{13.346^\circ}$$

$$\frac{\sin \hat{C}}{c} = \frac{\sin \hat{B}}{b} \Rightarrow \boxed{c} = \frac{b \sin \hat{C}}{\sin \hat{B}} = \boxed{1.80797 \text{ cm}}$$

