Recit #13
Apr 19, 2022

$\tilde{1}$ $a = 5$ cm, $b = 6$ cm, $c = 8$ cm. Exact, then round to 0.001.

$$s = \frac{5+6+8}{2} = \frac{19}{2}$$

[Heron's formula]

$$\text{Area}(T) = \sqrt{\frac{19}{2}\left(\frac{19}{2}-5\right)\left(\frac{19}{2}-6\right)\left(\frac{19}{2}-8\right)}$$

$$= \sqrt{\frac{19}{2}\cdot\frac{9}{2}\cdot\frac{7}{2}\cdot\frac{3}{2}}$$

$$\boxed{\begin{aligned} &= \frac{3}{4}\sqrt{19\cdot 7\cdot 3}\ \text{cm}^2 \\ &= \frac{3}{4}\sqrt{399}\ \text{cm}^2 \\ &\approx 14.9812\ \text{cm}^2 \end{aligned}}$$

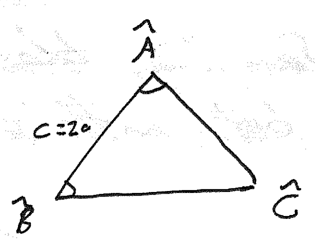
$\tilde{2}$ $a = 10$ cm, $\hat{B} = 60°$, $c = 12$ cm. Exact.

[SAS formula]

$$\text{Area}(T) = \frac{ac\sin\hat{B}}{2} = \frac{10\cdot 12\cdot \sin 60°}{2} = 60\cdot\frac{\sqrt{3}}{2}$$

$$= \boxed{30\sqrt{3}\ \text{cm}^2}$$

$\tilde{3}$ $\hat{A} = 32°$, $\hat{B} = 64°$, $c = 20$ cm. Round to 0.001



$$\hat{C} = 180° - 32° - 64° = 84°$$

$$\frac{\sin\hat{C}}{c} = \frac{\sin\hat{B}}{b} \Rightarrow b = c\,\frac{\sin\hat{B}}{\sin\hat{C}}$$

$$= 20\cdot\frac{\sin 64°}{\sin 84°} \approx 18.0749\ \text{cm}$$

$$\text{Area}(T) = \frac{bc\sin\hat{A}}{2} = \frac{18.0749\cdot 20\cdot\sin 32°}{2} \approx \boxed{95.7824\ \text{cm}^2}$$

4-5 Find exact values of sine, cosine, tangent, for the given angle, using sum/difference

$\theta = 15°$ $(= 45° - 30°)$

$$\sin(15°) = \sin(45° - 30°)$$
$$= \sin 45° \cos 30° - \sin 30° \cos 45°$$
$$= \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$
$$= \boxed{\frac{\sqrt{2}}{4}(\sqrt{3}-1)}$$

$$\cos(15°) = \cos(45° - 30°)$$
$$= \cos 45° \cos 30° + \sin 45° \sin 30°$$
$$= \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$= \boxed{\frac{\sqrt{2}}{4}(\sqrt{3}+1)}$$

$$\tan(15°) = \tan(45° - 30°)$$
$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$$
$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\cdot\frac{1}{\sqrt{3}}} = \boxed{\frac{\sqrt{3}-1}{\sqrt{3}+1}}$$

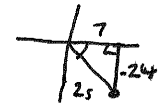
Or, if you rationalize the denominator:

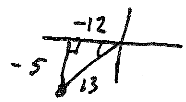
$$\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{3+1-2\sqrt{3}}{3-1}$$
$$= \boxed{2-\sqrt{3}}$$

6-9

Given α in Q IV with $\tan\alpha = \frac{-24}{7}$

β in Q III with $\sin\beta = \frac{-5}{13}$

α: 

β: 

Find sine, cosine, and tangent of $\gamma = \alpha + \beta$ as well as the quadrant of γ. Use <u>exact</u> values.

- $\sin(\gamma) = \sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$$= \frac{-24}{25}\cdot\frac{-12}{13} + \frac{-5}{13}\cdot\frac{7}{25}$$

$$= \boxed{\frac{253}{325}}$$

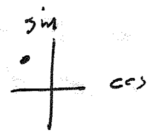
- $\cos(\gamma) = \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= \frac{7}{25}\cdot\frac{-12}{13} - \frac{-24}{25}\,\frac{-5}{13}$$

$$= \boxed{-\frac{204}{325}}$$

- $\tan(\gamma) = \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$$= \frac{\frac{-24}{7} + \frac{5}{12}}{1 - \left(\frac{-24}{7}\right)\left(\frac{5}{12}\right)} = \boxed{-\frac{253}{204}}$$



γ is in $\boxed{\text{Quad II}}$ since $\sin > 0$, $\cos < 0$

10 - 12

a) Find exact value

$$\cos\left[\operatorname{acos}\left(\frac{-2}{5}\right) - \operatorname{asin}\left(\frac{-3}{5}\right)\right]$$

Consider the general problem:

$$\cos[\underbrace{\operatorname{acos}(x)}_{\alpha} - \underbrace{\operatorname{asin}(y)}_{\beta}]$$

$$= \cos\alpha \cos\beta + \sin\alpha \sin\beta$$ [difference of cosines formula]

$$= \cos(\operatorname{acos} x) \cdot \cos(\operatorname{asin} y) + \sin(\operatorname{acos}(x)) \sin(\operatorname{asin} y)$$

$$= x \cdot \sqrt{1-y^2} + \sqrt{1-x^2} \cdot y$$

For the particular problem, $x = \frac{-2}{5}$, $y = -\frac{3}{5}$. Substitute in.

$$= -\frac{2}{5}\sqrt{1-\frac{9}{25}} + \sqrt{1-\frac{4}{25}} \cdot \frac{-3}{5}$$

$$= \frac{-2}{5} \cdot \frac{4}{5} + \frac{-3}{5} \cdot \frac{\sqrt{21}}{5}$$

$$= \boxed{-\frac{8}{25} + \frac{3\sqrt{21}}{25}}$$

b) Find exact value

$$\sin\left[\operatorname{acos}\left(\frac{-4}{5}\right) + \operatorname{atan}\left(\frac{12}{5}\right)\right]$$

Consider the general problem:

$$\sin\Big[\underbrace{\operatorname{acos}(x)}_{=\alpha} + \underbrace{\operatorname{atan}(y)}_{=\beta}\Big]$$

$$= \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

$$= \sin(\operatorname{acos} x)\cdot\cos(\operatorname{atan} y) + \sin(\operatorname{atan} y)\cdot\cos(\operatorname{acos} x)$$

$$= \sqrt{1-x^2}\cdot\frac{1}{\sqrt{1+y^2}} + \frac{y}{\sqrt{1+y^2}}\cdot x$$

For the particular problem, $x = \frac{-4}{5}$, $y = \frac{12}{5}$. Substitute in

$$= \sqrt{1-\frac{16}{25}}\cdot\frac{1}{\sqrt{1+\frac{144}{25}}} + \frac{12}{5}\cdot\frac{1}{\sqrt{1+\frac{144}{25}}}\cdot\frac{-4}{5}$$

$$= \frac{3}{\cancel{5}}\cdot\frac{\cancel{5}}{13} + \frac{12}{\cancel{5}}\cdot\frac{\cancel{5}}{13}\cdot\frac{-4}{5}$$

$$= \frac{3}{13} - \frac{48}{13\cdot 5} = \boxed{\frac{-33}{65}}$$

- $\sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$
- $\frac{1}{\sqrt{1+\frac{144}{25}}} = \frac{1}{\sqrt{\frac{169}{25}}} = \frac{1}{\frac{13}{5}} = \frac{5}{13}$