Recit #14
Apr 26, 2022

$\boxed{1-2}$ $\sqrt{3}\cos x - \sin x = -\sqrt{2}$

$$\tau = (-)\,\mathrm{acos}\left(\frac{\sqrt{3}}{\sqrt{3+1}}\right) = (-)\,\mathrm{acos}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{6}$$

$$\Rightarrow \sqrt{3+1}\,\cos(x-\tau) = -\sqrt{2}$$

$$\Rightarrow \cos\left(x+\frac{\pi}{6}\right) = \frac{-\sqrt{2}}{2}$$

$$\Rightarrow x+\frac{\pi}{6} = \frac{3\pi}{4}+2\pi k$$

$$\text{or } \frac{5\pi}{4}+2\pi k$$

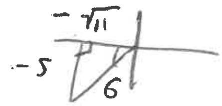
$$\Rightarrow x = \frac{3\pi}{4}-\frac{\pi}{6}+2\pi k \quad \text{or} \quad \frac{5\pi}{4}-\frac{\pi}{6}+2\pi k$$

$$= \boxed{\frac{7\pi}{12}+2\pi k \quad \text{or} \quad \frac{13\pi}{12}+2\pi k}$$

For 3-5, Calculate sin, cos and tan of 2θ, as well as quadrant. Exact values

$\tilde{3}$ $\sin\theta = \frac{-5}{6}$, θ in quadrant III

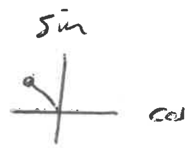
$\sqrt{36-25} = \sqrt{11}$



$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{-5}{6}\right)\frac{-\sqrt{11}}{6}$$
$$= \boxed{\frac{+5\sqrt{11}}{18}}$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$= 1 - 2\left(\frac{25}{36}\right)$$
$$= 1 - \frac{25}{18} = \boxed{\frac{-7}{18}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \boxed{\frac{5\sqrt{11}}{-7}}$$



Since sin > 0, cos < 0,

2θ is in $\boxed{Q\ II}$

4

$\tan\theta = 5$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{10}{26} = \boxed{\frac{5}{13}}$$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{-24}{26} = \boxed{\frac{-12}{13}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} \text{ or } \frac{2\tan\theta}{1-\tan^2\theta} = \boxed{\frac{-5}{12}}$$

Since $\sin 2\theta > 0$, $\cos 2\theta < 0$,

2θ is in $\boxed{Q\ II}$

5

$\theta = \text{asin}\left(\frac{1}{11}\right)$

$$\left[\cos(\text{asin}\, x) = \sqrt{1-x^2}\right]$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\cdot\frac{1}{11}\cdot\sqrt{1-\frac{1}{121}}$$
$$= \frac{2}{11}\cdot\sqrt{\frac{120}{121}}$$
$$= \frac{2}{121}\cdot\sqrt{120}$$

take 4 out of root.

$$= \frac{2\cdot 2\sqrt{30}}{121}$$
$$= \boxed{\frac{4\sqrt{30}}{121}}$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$= 1 - 2\left(\cancel{\sin(\text{asin}}\left(\frac{1}{11}\right))\right)^2$$
$$= 1 - 2\left(\frac{1}{11}\right)^2$$
$$= 1 - \frac{2}{121} = \boxed{\frac{-119}{121}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \boxed{-\frac{4\sqrt{30}}{119}}$$

$\sin > 0$, $\cos < 0$

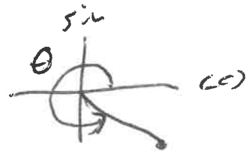
2θ is in $\boxed{Q\ II}$

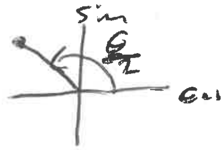
For 6–8 Find sin, cos, tan of $\frac{\theta}{2}$. Exact.

6

$\cos\theta = \frac{2}{3}$, $\pi < \theta < 2\pi$

$\frac{\pi}{2} < \frac{\theta}{2} < \pi$





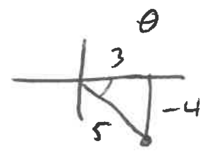
$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} = +\sqrt{\frac{1-\frac{2}{3}}{2}} = \boxed{+\sqrt{\frac{1}{6}}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+\frac{2}{3}}{2}} = \boxed{-\sqrt{\frac{5}{6}}}$$

$$\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{\sqrt{\frac{1}{6}}}{-\sqrt{\frac{5}{6}}} = -\sqrt{\frac{\frac{1}{6}}{\frac{5}{6}}} = \boxed{-\sqrt{\frac{1}{5}}}$$

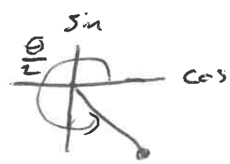
7

$\tan\theta = \frac{-4}{3}$ $\quad 3\pi < \theta < 4\pi$



$\Rightarrow \cos\theta = \frac{3}{5}$

$\frac{3\pi}{2} < \frac{\theta}{2} < 2\pi$



$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} = -\sqrt{\frac{1-\frac{3}{5}}{2}} = -\sqrt{\frac{\frac{2}{5}}{2}} = -\sqrt{\frac{1}{5}} = \boxed{-\frac{1}{\sqrt{5}}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} = +\sqrt{\frac{1+\frac{3}{5}}{2}} = +\sqrt{\frac{\frac{8}{5}}{2}} = +\sqrt{\frac{4}{5}} = \boxed{\frac{2}{\sqrt{5}}}$$

$$\tan\frac{\theta}{2} = \frac{\sin}{\cos} = \boxed{\frac{-1}{2}}$$

8 is similar to 6 & 7.

9–10) Find the Fourier series

$$F(t) = 4\,\underbrace{\sin(t)\sin(2t)}\,\sin(4t)$$

$$= 4\cdot\tfrac{1}{2}\left[\cos(-t) - \cos(3t)\right]\cdot\sin(4t)$$

$$= 2\,\underbrace{\cos(t)\sin 4t} - 2\,\underbrace{\cos(3t)\sin(4t)}$$

$$= \cancel{2}\cancel{\tfrac{1}{2}}\left[\sin(5t)+\sin(3t)\right] - \cancel{2}\cancel{\tfrac{1}{2}}\left[\sin(7t)+\sin(t)\right]$$

$$= -\sin(t) + \sin(3t) + \sin(5t) - \sin(7t)$$

11–13) Find all solutions to the eqn. Exact values. Hint: Sum-to-Product

$$\sin(7x) + \sin(3x) = 0$$

$$\leadsto 2\sin\left(\frac{7x+3x}{2}\right)\cos\left(\frac{7x-3x}{2}\right)$$

$$= 2\sin(5x)\cos(2x) = 0$$

$$\Rightarrow \sin 5x = 0 \qquad\Big|\qquad \cos(2x) = 0$$

$$2x = \frac{\pi}{2} + \pi k$$

$$\Rightarrow \boxed{x = \frac{\pi}{4} + \frac{\pi}{2}k}$$

$$5x = 0 + \pi k$$

$$\Rightarrow \boxed{x = \frac{\pi}{5}k}$$

14 - 15

$$\frac{\cos 6y + \cos 8y}{\sin 6y - \sin 4y} = \cot y \cos 7y \sec 5y$$

$$= \frac{\cancel{2} \cos(7y)\cos(-y)}{\cancel{2} \sin(y)\cos(5y)}$$

$$= \cos(7y)\cot(y)\sec(5y) \checkmark$$