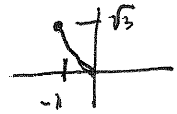
$\widetilde{1-3}$

Find all polar repr. Indicate principal one.

Recit #15
May 3, 2022

a)

$$z = -1 + \sqrt{3}\,i$$



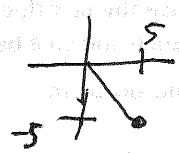

$$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$$

$$\theta = (\text{sign}\ \sqrt{3}) \cdot \operatorname{acos}\left(\frac{-1}{2}\right)$$

$$= (+) \cdot \frac{2\pi}{3}$$

$$\boxed{\begin{array}{l} \text{All reprs: } 2\ \text{cis}\left(\frac{2\pi}{3} + 2\pi k\right) \\ \text{Principal: } 2\ \text{cis}\left(\frac{2\pi}{3}\right) \end{array}}$$

b) $z = 5 - 5i$




$$r = \sqrt{5^2 + (-5)^2} = 5\sqrt{2}$$

$$\theta = (\text{sign}(-5)) \cdot \operatorname{acos}\left(\frac{\cancel{5}}{\cancel{5}\sqrt{2}}\right)$$

$$= (-)\ \frac{\pi}{4}$$

$$\boxed{\begin{array}{l} \text{All repr: } 5\sqrt{2}\ \text{cis}\left(-\frac{\pi}{4} + 2\pi k\right) \\ \text{Principal repr: } 5\sqrt{2}\ \text{cis}\left(-\frac{\pi}{4}\right) \end{array}}$$

4-6

a) $8\,\mathrm{cis}\left(-\frac{3\pi}{4}\right) = 8\left[\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right]$

$= 8\left[-\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right]$

$= \boxed{-4\sqrt{2} - 4\sqrt{2}\,i}$

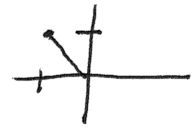
b) $5\,\mathrm{cis}\left(-\frac{\pi}{6}\right) = 5\left[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right]$

$= 5\left[\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right]$

$= \boxed{\frac{5\sqrt{3}}{2} - \frac{5}{2}\,i}$

7-9

a) $(-1+i)^6 = \left(\sqrt{2}\ \text{cis}\ \frac{3\pi}{4}\right)^6$



$r = \sqrt{2}$

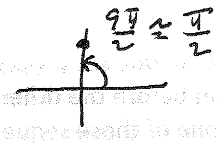
$\theta = \frac{3\pi}{4}$

$= 2^3\, \text{cis}\left(\frac{9\pi}{2}\right)$

$\frac{9\pi}{2} \simeq \frac{\pi}{2}$

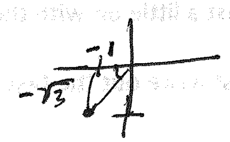
$= 8\left[\cos\left(\frac{9\pi}{2}\right) + i\sin\left(\frac{9\pi}{2}\right)\right]$

$\quad\ \ 0 \quad + 1i$




$= \boxed{8i}$

b) $(-1-\sqrt{3}i)^7$




$= \left[2\ \text{cis}\left(-\frac{2\pi}{3}\right)\right]^7$

$r = \sqrt{1+3} = 2$

$\theta = (-)\cdot a\cos\left(\frac{-1}{2}\right) = -\frac{2\pi}{3}$

$= 2^7\ \text{cis}\left(-\frac{14\pi}{3}\right)$

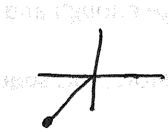
$= 128\left[\cos\left(-\frac{14\pi}{3}\right) + i\sin\left(-\frac{14\pi}{3}\right)\right]$

$-\frac{14\pi}{3} + \frac{6\pi}{3}\cdot 3 = \frac{-14\pi + 18\pi}{3}$

$= \frac{4\pi}{3}$

$= 128\left[-\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right]$

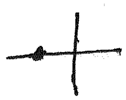
$= \boxed{-64 - 64\sqrt{3}\,i}$




$60°$ in III

10-12

a) 4th roots of $-81$

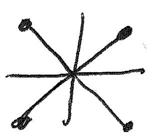


$$w = -81 = 81 \text{ cis } \pi$$

$$z_k = \sqrt[4]{w} = \sqrt[4]{81} \text{ cis}\left(\frac{\pi + 2\pi k}{4}\right) \qquad k = 0, 1, 2, 3$$

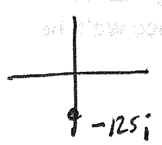
$$= 3 \text{ cis}\left(\frac{\pi}{4} + \frac{\pi}{2}k\right)$$

$$= 3 \text{ cis } \frac{\pi}{4},\ 3 \text{ cis } \frac{3\pi}{4},\ 3 \text{ cis } \frac{5\pi}{4},\ 3 \text{ cis } \frac{7\pi}{4}$$



$$= \boxed{\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i,\quad -\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i,\quad -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i,\quad \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i}$$

b) 3rd roots of $-125i$




$$w = -125i = 125 \text{ cis } \frac{3\pi}{2}$$

$$z_k = \sqrt[3]{w} = \sqrt[3]{125} \text{ cis}\left(\frac{\pi}{2} + \frac{2\pi k}{3}\right) \qquad k = 0, 1, 2$$

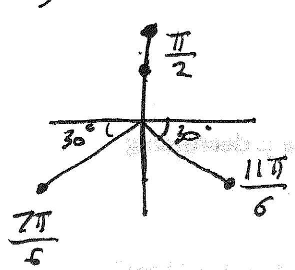
$$= 5 \text{ cis}\left(\frac{\pi}{2}\right),\ 5 \text{ cis}\left(\frac{7\pi}{6}\right),\ 5 \text{ cis}\left(\frac{11\pi}{6}\right)$$

$$\frac{\pi}{2} + \frac{2\pi}{3} = \frac{4\pi + 3\pi}{6} = \frac{7\pi}{6}$$

$$= 5(0 + i),\ 5\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right),\ 5\left(+\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$$

$$\frac{\pi}{2} + \frac{4\pi}{3} = \frac{8\pi + 3\pi}{6} = \frac{11\pi}{6}$$

$$= \boxed{5i,\ -\frac{5\sqrt{3}}{2} - \frac{5}{2}i,\ \frac{5\sqrt{3}}{2} - \frac{5}{2}i}$$



14) $x^8 - 97x^4 + 1296 = 0$

Let $z = x^4$

$1296 = 2^4 \cdot 3^4$

$16 + 81 = 97$

$z^2 - 97z + 1296 = 0$

$(z-81)(z-16) = 0$

$(x^4-81)(x^4-16) = 0$

$x^4 = 81$ or $x^4 = 16$

$= 3^4 \operatorname{cis} 0$

$= 2^4 \operatorname{cis} 0$

$x = 3 \operatorname{cis} \frac{2\pi k}{4}$

$= 3,\ 3\operatorname{cis}\frac{\pi}{2},\ 3\operatorname{cis}\pi,\ 3\operatorname{cis}\frac{3\pi}{2}$

$= \boxed{3,\ 3i,\ -3,\ -3i}$

$x = 2 \operatorname{cis}\left(0 + \frac{2\pi k}{4}\right)$

$= 2,\ 2\operatorname{cis}\frac{\pi}{2},\ 2\operatorname{cis}\pi,\ 2\operatorname{cis}\frac{3\pi}{2}$

$= \boxed{2,\ 2i,\ -2,\ -2i}$