TRIGONOMETRY HOMEWORK 3

1. Find the equation of the circle centered at $Z(2,3)$, which passes through the point $P(-4,7)$.

2. Find the center and the radius of the circle represented by the equation $4x^2 + (2y-1)^2 = 9$.

3. Using bearing notation, find the two directions that are *perpendicular* to N76°W.

4. Find an equation for the line which passes through $P(4,2)$ and $Q(1,-2)$.

5. Given the point $P(-1,1)$ and the line $\mathscr{L}$ with equation

$$7x - 3y = 8,$$

find the equations of the following lines:

(i) the line $\mathscr{L}'$ which passes through P and is *parallel* to $\mathscr{L}$;

(ii) the line $\mathscr{L}''$ which passes through P and is *perpendicular* to $\mathscr{L}$

6. Given the vectors $\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 8 \\ -10 \end{bmatrix}$, find the following vectors in coordinates and their magnitudes (use **exact** values and assume the length is measured in inches):

(i) $2\vec{u} + 3\vec{v}$;

(ii) $3\vec{u} - 2\vec{v}$.

In Exercises 7–8, find all six trigonometric functions of θ, based on the fact that a certain point A sits on the terminal side of its corresponding standard position angle. Use **exact** values!

7. $A(3,-4)$.

8. $A(-7,24)$.

In Exercises 9–12, find the **exact** values of all six trigonometric functions for the given angle θ. If any value is *undefined*, state so.

9. $\theta = -690°$

10. $\theta = 765°$

11. $\theta = 123\pi$

12. $\theta = -\dfrac{15\pi}{2}$

Recit #5
Feb 15, 2022

$\tilde{1}$ Eqn of circle centered at $(4,-1)$ passing through $(2,3)$

Need radius (distance b/t these given points)

Formula for circle of radius r centered at (h,k):

$$(x-h)^2+(y-k)^2=r^2$$

$$r=\sqrt{(4-2)^2+(-1-3)^2}$$
$$=\sqrt{2^2+(-4)^2}$$
$$=\sqrt{4+16}$$
$$=\sqrt{20}=2\sqrt{5}$$

Another approach: Can plug both points into the equation of a circle to find the radius:

$$r^2=(2-4)^2+(3-(-1))^2$$
$$=(-2)^2+4^2$$
$$=4+16=20$$

so $\boxed{(x-4)^2+(y+1)^2=20}$ is the equation for this circle

$\tilde{2}$ Find center and radius of circle represented by

$$9x^2+(3y-2)^2=25$$

want to get rid of this 3 to match the formula of a circle

$$\Rightarrow\ 9x^2+\left[3\left(y-\tfrac{2}{3}\right)\right]^2=25$$

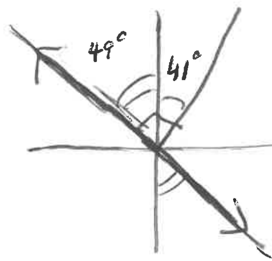
$$\Rightarrow\ 9x^2+9\left(y-\tfrac{2}{3}\right)^2=25$$

$$\Rightarrow\ x^2+\left(y-\tfrac{2}{3}\right)^2=\tfrac{25}{9}$$

$\therefore$ $\boxed{\text{center}: \left(0,\tfrac{2}{3}\right) \quad \text{radius}: \tfrac{5}{3}}$

$\tilde{3}$ Using bearing notation, find the two directions perpendicular to N 41° E

$$90° - 41° = 49°$$



$\boxed{\text{Answer: N 49° W, S 49° E}}$

$\tilde{4}$ Find eqn of line passing through (1, 6) and (5, −3)

$$m = \frac{-3-6}{5-1} = \frac{-9}{4}$$

$$\boxed{y - 6 = \frac{-9}{4}(x-1)}$$ ← this is point-slope form $y - y_0 = m(x_1 - x_0)$

$\tilde{5}$ Given the point $P(4,-5)$ and line $\mathcal{L}$: $2x - 7y = 12$ $\Rightarrow$ $2x - 12 = 7y$

$\Rightarrow \frac{2}{7}x - \frac{12}{7} = y$

need this.

i) Find line $\mathcal{L}'$ parallel to $\mathcal{L}$ and passing through P

use same slope

$$\boxed{\mathcal{L}': y + 5 = \frac{2}{7}(x-4)}$$

ii) Find line $\mathcal{L}''$ perpendicular to $\mathcal{L}$ and passing through P

use negative reciprocal of slope

$$\boxed{\mathcal{L}'': y + 5 = -\frac{7}{2}(x-4)}$$

$\boxed{6}$ Given $\vec{u} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$,

find the following vectors and their magnitudes

(use <u>exact</u> values; assume length is in inches)

i) $2\vec{u} + 3\vec{v}$

$$= 2\begin{bmatrix} -4 \\ 3 \end{bmatrix} + 3\begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -8 \\ 6 \end{bmatrix} + \begin{bmatrix} 15 \\ 6 \end{bmatrix} = \boxed{\begin{bmatrix} 7 \\ 12 \end{bmatrix}}$$

$$\|2\vec{u} + 3\vec{v}\| = \sqrt{7^2 + 12^2} = \sqrt{49 + 144}$$

$$= \boxed{\sqrt{193} \text{ in.}}$$

ii) $3\vec{u} - 2\vec{v} = 3\begin{bmatrix} -4 \\ 3 \end{bmatrix} - 2\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

$$= \begin{bmatrix} -12 \\ 9 \end{bmatrix} - \begin{bmatrix} 10 \\ 4 \end{bmatrix} = \boxed{\begin{bmatrix} -22 \\ 5 \end{bmatrix}}$$

$$\|3\vec{u} - 2\vec{v}\| = \sqrt{(-22)^2 + 5^2}$$

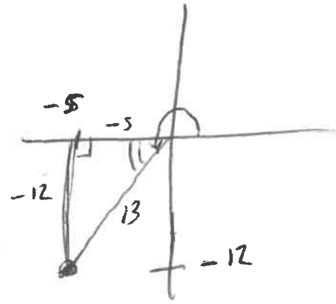
$$= \boxed{\sqrt{509}}$$

$\tilde{7}$ Find all six trig function of θ ....

Exact values

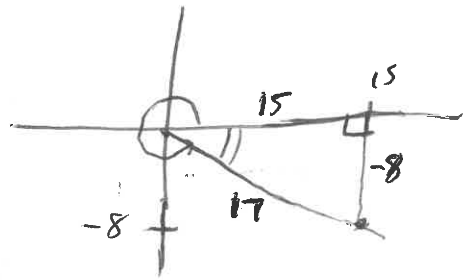
$A = (-5, -12)$

$$\boxed{\begin{array}{ll} \sin\theta = \frac{-12}{13} & \csc\theta = \frac{-13}{12} \\ \cos\theta = \frac{-5}{13} & \sec\theta = \frac{-13}{5} \\ \tan\theta = \frac{-12}{-5} = \frac{12}{5} & \cot\theta = \frac{5}{12} \end{array}}$$



$\tilde{8}$ $A(15, -8)$

$$\boxed{\begin{array}{ll} \sin\theta = \frac{-8}{17} & \csc\theta = \frac{-17}{8} \\ \cos\theta = \frac{15}{17} & \sec\theta = \frac{17}{15} \\ \tan\theta = \frac{-8}{15} & \cot\theta = \frac{-15}{8} \end{array}}$$



$\tilde{9}$) $\theta = -1380^\circ$

$-1380^\circ + 4\cdot 360^\circ = 60^\circ$

$\boxed{\text{Idea: Add multiple copies of } 360^\circ \text{ until get an angle b/t } 0^\circ \text{ and } 360^\circ}$

So

these angles (-1380° and 60°) are equivalent.

[The 6 trig functions will return the same value when you plug in these two angles.]

Just need to know values for $\theta = 60^\circ$. (unit circle)

$$\boxed{\begin{array}{ll} \sin\theta = \frac{\sqrt{3}}{2} & \csc\theta = \frac{2}{\sqrt{3}} \\ \cos\theta = \frac{1}{2} & \sec\theta = 2 \\ \tan\theta = \sqrt{3} & \cot\theta = \frac{1}{\sqrt{3}} \end{array}}$$

$\tilde{10}$) $\theta = 405^\circ$

This time, subtract multiples of 360°.

$405^\circ - 1\cdot 360^\circ = 45^\circ$

$$\boxed{\begin{array}{ll} \sin\theta = \frac{\sqrt{2}}{2} & \csc\theta = \sqrt{2} \\ \cos\theta = \frac{\sqrt{2}}{2} & \sec\theta = \sqrt{2} \\ \tan\theta = 1 & \cot\theta = 1 \end{array}}$$

$\boxed{\widetilde{11}}$ $\theta = \frac{-13\pi}{2}$

Working in radians, we add / subtract multiples of 2π to get equivalent angles.

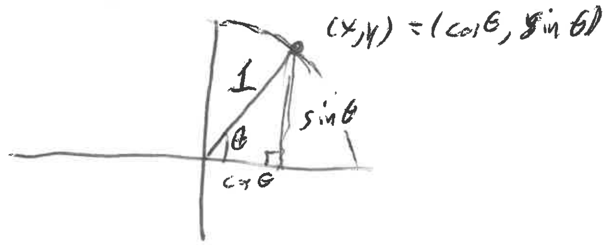
$$\frac{-13\pi}{2} + 3 \cdot 2\pi = \frac{-\pi}{2}$$

← this is the angle $-\frac{\pi}{2}$ visually

$$\boxed{\begin{array}{ll} \sin\theta = -1 & \csc\theta = -1 \\ \cos\theta = 0 & \sec\theta = \text{undef} \\ \tan\theta = \text{undef} & \cot\theta = 0 \end{array}}$$

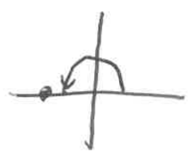
Recall that $\cos\theta$ is x-coord and $\sin\theta$ is y-coord

On unit circle:



$\boxed{\widetilde{12}}$ $\theta = 7\pi$

$$7\pi - 3 \cdot 2\pi = \pi$$



$$\boxed{\begin{array}{ll} \sin\theta = 0 & \csc\theta = \text{undef} \\ \cos\theta = -1 & \sec\theta = -1 \\ \tan\theta = 0 & \cot\theta = \text{undef} \end{array}}$$