TRIGONOMETRY HOMEWORK 4

In Exercises 1–4, find the values of all remaining five trigonometric functions of the angle θ, based on the given quadrant information. Use **exact** values!

1. $\sin\theta = -\frac{3}{5}$, θ in quadrant III.

2. $\cos\theta = -\frac{2}{7}$, θ in quadrant II.

3. $\cot\theta = -\frac{5}{9}$, θ in quadrant IV.

4. $\csc\theta = \sqrt{10}$, θ in quadrant II.

5. Assume $\cos\theta = -\frac{99}{101}$ and $\sin\theta = \frac{20}{101}$. Find the **exact** values of all six trigonometric functions, as well as the quadrant location, for each one of the following:

(a) $-\theta$;

(b) $\pi + \theta$;

(c) $\pi - \theta$.

In Exercises 6–9, find the **exact** values of all six trigonometric functions for the given angle θ. If any value is *undefined*, state so.

6. $\theta = -585°$

7. $\theta = 870°$

8. $\theta = \frac{5\pi}{4}$

9. $\theta = -\frac{2\pi}{3}$

In Exercises 10-13, you are asked to verify the given identity.

10. $\sec t - \cos t = \tan t \sin t$.

11. $\cos x + \sin x \tan x = \sec x$.

12. $(\tan s + \cot s)(\sin s + \cos s) = \sec s + \csc s$.

13. $\frac{1}{1 - \sin w} + \frac{1}{1 + \sin w} = 2\sec^2 w$.

In each one of Exercises 14-15, you are given a certain equality, which you are asked to show that is a false identity, so you need to find one value for the variable, for which the equality is not true.

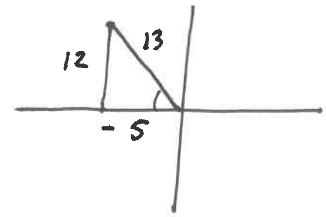
14. $(\sin x + \cos x)^3 = \sin^3 x + \cos^3 x$.

15. $\tan(2x) = 2\tan x$.

Recit #6
Feb 22, 2022

1–4 Find the remaining trig functions on the angle θ.
Use ~~the~~ exact values

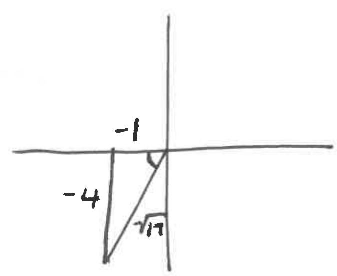
a) $\tan\theta = \frac{-12}{5}$, quadrant II



$$\boxed{\begin{array}{ll} \sin\theta = \frac{12}{13} & \csc\theta = \frac{13}{12} \\ \cos\theta = \frac{-5}{13} & \sec\theta = \frac{-13}{5} \\ \tan\theta = -\frac{12}{5} & \cot\theta = \frac{-5}{12} \end{array}}$$

Draw representative triangle, then read off values.

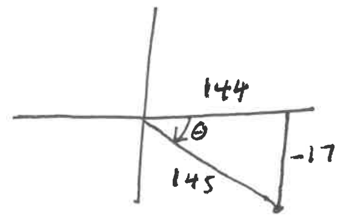
b) $\sec\theta = -\sqrt{17}$, quadrant III



$$\boxed{\begin{array}{ll} \sin\theta = \frac{-4}{\sqrt{17}} & \csc\theta = \frac{-\sqrt{17}}{4} \\ \cos\theta = \frac{-1}{\sqrt{17}} & \sec\theta = -\sqrt{17} \\ \tan\theta = 4 & \cot\theta = \frac{1}{4} \end{array}}$$

$\hat{5}$ Assume $\sin\theta = \frac{-17}{145}$, $\cos\theta = \frac{144}{145}$.

Find exact value of all six trig functions, as well as quadrant location



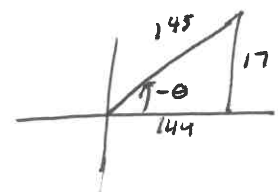
a) $-\theta$

$$\sin(-\theta) = \frac{17}{145} \qquad \csc(-\theta) = \frac{145}{17}$$

$$\cos(-\theta) = \frac{144}{145} \qquad \sec(-\theta) = \frac{145}{144}$$

$$\tan(-\theta) = \frac{17}{144} \qquad \cot(-\theta) = \frac{144}{17}$$

Quadrant I



Can also use even/odd properties of sine and cosine.

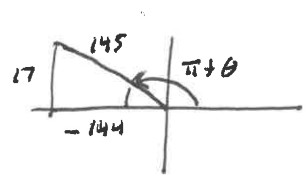
b) $\pi + \theta$

$$\sin(\pi+\theta) = \frac{17}{145} \qquad \csc(\pi+\theta) = \frac{145}{17}$$

$$\cos(\pi+\theta) = \frac{-144}{145} \qquad \sec(\pi+\theta) = -\frac{145}{144}$$

$$\tan(\pi+\theta) = -\frac{17}{144} \qquad \cot(\pi+\theta) = \frac{-144}{17}$$

Quadrant II



Can also use "Add π formulas"

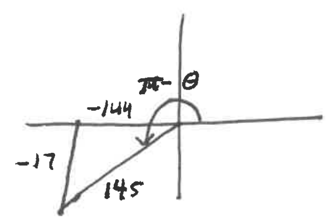
c) $\pi - \theta$

$$\sin(\pi-\theta) = \frac{-17}{145} \qquad \csc(\pi-\theta) = \frac{-145}{17}$$

$$\cos(\pi-\theta) = \frac{-144}{145} \qquad \sec(\pi-\theta) = \frac{-145}{144}$$

$$\tan(\pi-\theta) = \frac{17}{144} \qquad \cot(\pi-\theta) = \frac{144}{17}$$

Quadrant III

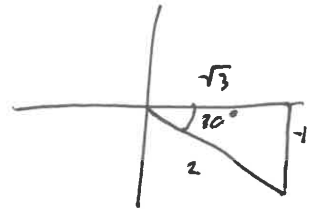


Can also use supplement formulas

6-7

$\theta = -1110°$ $(+3\cdot 360°)$

$\simeq -30°$



$$\boxed{\begin{array}{ll} \sin\theta = -\frac{1}{2} & \csc\theta = -2 \\ \cos\theta = \frac{\sqrt{3}}{2} & \sec\theta = \frac{2}{\sqrt{3}} \\ \tan\theta = -\frac{1}{\sqrt{3}} & \cot\theta = -\sqrt{3} \end{array}}$$

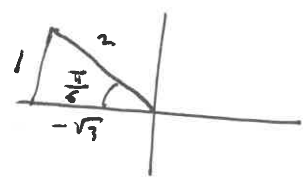
Think about the trig functions at the reference angle (30° for this problem)

Then determine the correct sign based on the quadrant the angle is in.

8-9

$\theta = \frac{17\pi}{6}$

$\simeq \frac{5\pi}{6}$



$$\boxed{\begin{array}{ll} \sin\theta = \frac{1}{2} & \csc\theta = 2 \\ \cos\theta = -\frac{\sqrt{3}}{2} & \sec\theta = -\frac{2}{\sqrt{3}} \\ \tan\theta = -\frac{1}{\sqrt{3}} & \cot\theta = -\sqrt{3} \end{array}}$$

$\boxed{10-13}$ Verify identities

- Practice
- Try breaking / rewriting things into sin & cos.
- Remember - reciprocal identities
  - Pythagorean identities
- Algebraic manipulation

a) $\dfrac{\sec\theta}{\cos\theta} - \dfrac{\tan\theta}{\cot\theta} = 1$

$= \dfrac{1}{\cos^2\theta} - \tan^2\theta$

$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$= \dfrac{1-\sin^2\theta}{\cos^2\theta} = \dfrac{\cos^2\theta}{\cos^2\theta} = 1$ ✓

b) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

$= \sin^2\theta + \cancel{2\sin\theta\cos\theta} + \cos^2\theta$
$+ \sin^2\theta - \cancel{2\sin\theta\cos\theta} + \cos^2\theta$

$= 2\sin^2\theta + 2\cos^2\theta$

$= 2(\sin^2\theta + \cos^2\theta) = 2$ ✓

c) $\dfrac{1-\tan^2 x}{1+\tan^2 x} = 1 - 2\sin^2 x$

$1 + \tan^2 x = \sec^2 x$

$= \dfrac{1-\tan^2 x}{\sec^2 x}$

$= \cos^2 x - \dfrac{\tan^2 x}{\sec^2 x}$

$= \cos^2 x - \dfrac{\sin^2 x}{\cancel{\cos^2}} \cdot \cancel{\cot x} = \cos^2 x - \sin^2 x$

$= (1-\sin^2 x) - \sin^2 x$ ✓

d) $\dfrac{\cos x + 1}{\sin^3 x} = \dfrac{\csc x}{1 - \cos x}$

$$= \frac{\cos x + 1}{\sin x\,(1 - \cos^2 x)}$$

use difference of squares

$x^2 - a^2 = (x-a)(x+a)$

$$= \frac{1}{\sin x\,(1 - \cos x)}$$

$$= \frac{\csc x}{1 - \cos x} \quad ✓$$

e) $\dfrac{\tan\theta}{\sec\theta} + \dfrac{\cot\theta}{\csc\theta} = \sin\theta + \cos\theta$

← Algebraic manipulation

$$\frac{\frac{A}{B}}{\frac{C}{B}} \cdot \frac{B}{B} = \frac{A}{C}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$$

$$= \sin\theta + \cos\theta \quad ✓$$

f) $\dfrac{\sin x + \tan x}{1 + \sec x} = \sin x$

$$= \frac{\sin x + \frac{\sin x}{\cos x}}{1 + \frac{1}{\cos x}}$$

$$= \frac{\sin x \cos x + \sin x}{\cos x + 1}$$

$$= \frac{\sin x\,(\cancel{\cos x + 1})}{\cancel{\cos x + 1}} = \sin x \quad ✓$$

$\boxed{\widetilde{14}}$ Show the identity is false

$$(\sin x + \cos x)^4 = \sin^4 x + \cos^4 x$$

Let's try $x = \frac{\pi}{4}$ ← Pick an easy-to-use value that you think will work.

Evaluating the LHS:

$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

so $LHS = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)^4 = (\sqrt{2})^4 = 2^2 = \boxed{4}$

Evaluating the RHS:

$$\sin^4 x = \left(\frac{\sqrt{2}}{2}\right)^4 = \left(\frac{1}{\sqrt{2}}\right)^4 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$\cos^4 x$ same

so $RHS = \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2}}$

Thus $LHS \neq RHS$ for $x = \frac{\pi}{4}$.

15 Show the identity is false

$\sin(2x) = 2\sin x$

Let's try $x = \frac{\pi}{6}$ ← Pick an easy to use value that you think will work.

Evaluating the LHS:

$$\sin\left(2\cdot\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{2}}$$

Evaluating the RHS:

$$2\sin\left(\frac{\pi}{6}\right) = 2\cdot\frac{1}{2} = \boxed{1}$$

Thus LHS $\neq$ RHS for $x = \frac{\pi}{6}$.